Metric Analysis of Tapping Accuracy for Triple Meter in Chopin Mazurkas

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This *Mathematica* notebook examines the accuracy at which each type of metrical beat is tapped when reverse conducting to particular performances of Chopin Mazurkas. The tapping accur four performances of two mazurkas are analyzed below:

(1) Mazurka in A minor, Op. 7, No. 2 performed by Frederic Chiu in 1999
(2) Mazurka in A minor, Op. 7, No. 2 performance by Ignaz Friedman in 1930
(3) Mazurka in F minor, Op. 7, No. 3 performed by Ignaz Friedman in 1930
(4) Mazurka in F minor, Op. 7, No. 3 performed by Charles Rosen in 1989

Necessary packages to load first:

```
(* needed for Histogram function *)
<< Graphics`Graphics`

(* needed for DispersionReport and Mean functions *)
<< Statistics`ConfidenceIntervals`

Off[General::spell1] (* not interested in possible spelling errors *)
```

■ Tapping accuracy for each beat (20-trial average)

■ Mazurka in A minor, Op. 7, No. 2; Chiu 1999

```
pid9048x06corrected = {1725, 2529, 3219, 3794, 4355, 4843, 5522, 6246, 6755, 7217, 7758, 8190, 8831, 9591, 10061, 10563, 11048, 11566, 12103,
   12602, 13150, 13608, 14132, 14944, 16088, 16721, 17343, 17843, 18371, 18901, 19550, 20219, 20756, 21179, 21737, 22180, 22843, 23543,
   24137, 24646, 25205, 26008, 26891, 27478, 28066, 28603, 29193, 30021, 30874, 31605, 32214, 32766, 33313, 33803, 34446, 35112, 35638,
   36096, 36721, 37204, 37872, 38548, 38985, 39467, 39959, 40558, 41192, 41753, 42307, 42809, 43496, 44351, 45401, 46086, 46574, 47129,
   47681, 48145, 48715, 49410, 49916, 50354, 50899, 51385, 52032, 52666, 53167, 53664, 54284, 55110, 55975, 57093, 57641, 58142, 58696,
   59436, 60156, 60713, 61150, 61809, 62350, 62819, 63387, 64024, 64507, 65236, 65811, 66293, 66839, 67513, 67993, 68530, 69097, 69588,
   70080, 70718, 71220, 71886, 72625, 73197, 73989, 74683, 75278, 75799, 76336, 76793, 77395, 78052, 78557, 79030, 79493, 79932, 80542,
   81218, 81786, 82304, 82861, 83651, 84533, 85348, 85924, 86425, 87071, 87932, 88690, 89481, 89986, 90776, 91337, 91802, 92481,
   93194, 93658, 94272, 94814, 95282, 95928, 96722, 97295, 97949, 98691, 99247, 99802, 100569, 101150, 101974, 102687, 103302, 104211,
   104986, 105568, 106095, 106644, 107160, 107851, 108480, 109039, 109489, 109949, 110364, 111000, 111616, 112140, 112584, 113180,
   113964, 114829, 115705, 116544, 117219, 118034, 118810, 119532, 120311, 120886, 121404, 121864, 122382, 122864, 123262, 123697,
   124134, 124709, 125090, 125542, 125953, 126362, 126791, 127312, 127700, 128129, 128538, 128933, 129339, 129855, 130267, 130861,
   131575, 132079, 132579, 133010, 133440, 133952, 134359, 134789, 135267, 135696, 136138, 136658, 137334, 137774, 138271, 138679,
   139085, 139580, 140153, 140667, 141393, 142228, 143006, 143778, 144520, 145237, 145703, 146201, 146695, 147118, 147500, 147922,
   148358, 148883, 149259, 149671, 150070, 150472, 150918, 151311, 151725, 152121, 152526, 152937, 153391, 153921, 154369, 154888,
   155525, 156037, 156488, 156892, 157281, 157724, 158145, 158568, 158985, 159491, 159932, 160386, 161047, 161574, 162061, 162542,
   162958, 163450, 164036, 164617, 165353, 165946, 166720, 167489, 168388, 169005, 169509, 169997, 170467, 170905, 171324, 171771,
   172164, 172693, 173141, 173559, 173956, 174422, 174966, 175447, 175853, 176277, 176675, 177128, 177537, 178152, 178892, 180643,
   181165, 181685, 182174, 182941, 183411, 184108, 184940, 185395, 185814, 186306, 186787, 187476, 188303, 188750, 189235, 189682,
   190263, 191033, 191567, 192116, 192723, 193514, 194441, 195256, 196098, 196749, 197417, 198021, 198611, 199326, 200020, 200639,
   201096, 201810, 202487, 203397, 204373, 204899, 205423, 206038, 206783, 207781, 208869, 209460, 210070, 210895, 211945, 213210};
```

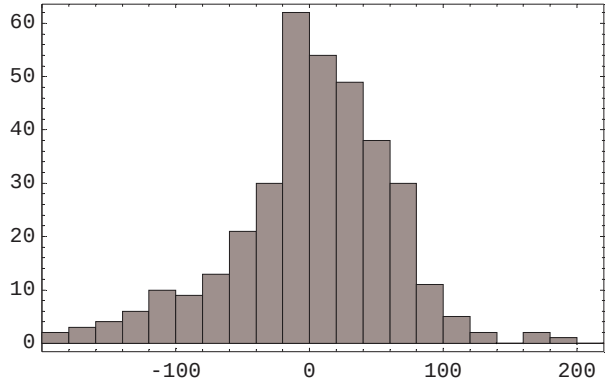
```
pid9048x06avg = {1912, 2557, 3174, 3775, 4431, 4913, 5542, 6289, 6802, 7217, 7813, 8307, 8934, 9553, 10056, 10543, 11059, 11613, 12176, 12677,
  13203, 13668, 14216, 14944, 15960, 16736, 17276, 17865, 18400, 18876, 19565, 20294, 20770, 21229, 21684, 22212, 22872, 23605, 24086,
  24621, 25173, 25930, 26891, 27682, 28118, 28617, 29206, 29980, 31020, 31585, 32209, 32766, 33360, 33856, 34510, 35212, 35685, 36142,
  36667, 37280, 37932, 38597, 39023, 39499, 40011, 40520, 41182, 41784, 42347, 42844, 43457, 44315, 45417, 46144, 46653, 47135, 47712,
  48224, 48822, 49389, 49913, 50387, 50854, 51390, 52040, 52693, 53167, 53647, 54198, 55001, 55975, 56847, 57688, 58207, 58757, 59469,
  60274, 60758, 61159, 61701, 62368, 62864, 63492, 64059, 64500, 65084, 65819, 66319, 66922, 67488, 68031, 68586, 69124, 69608, 70139,
  70646, 71256, 71819, 72520, 73243, 73944, 74696, 75245, 75831, 76374, 76855, 77457, 78120, 78578, 79082, 79543, 80013, 80584,
  81244, 81742, 82290, 82884, 83543, 84533, 85412, 85973, 86478, 87024, 87843, 88834, 89348, 89972, 90593, 91352, 91843, 92461,
  93180, 93660, 94256, 94860, 95329, 95905, 96653, 97275, 97908, 98614, 99247, 99868, 100480, 101144, 101812, 102711, 103329, 104114,
  104986, 105559, 106126, 106714, 107205, 107851, 108585, 109032, 109535, 109989, 110477, 111083, 111735, 112140, 112606, 113148,
  113917, 114829, 115680, 116326, 117105, 117932, 118772, 119524, 120280, 120913, 121381, 121884, 122355, 122864, 123301, 123697,
  124161, 124611, 125111, 125542, 125964, 126362, 126840, 127276, 127708, 128160, 128562, 128962, 129409, 129818, 130282, 130795,
  131511, 132122, 132579, 133088, 133454, 133927, 134443, 134853, 135267, 135768, 136128, 136645, 137334, 137866, 138276, 138759,
  139106, 139572, 140123, 140734, 141393, 142071, 143053, 143888, 144595, 145152, 145733, 146208, 146704, 147187, 147578, 147974,
  148402, 148867, 149319, 149729, 150112, 150494, 150924, 151399, 151750, 152167, 152552, 152952, 153414, 153914, 154425, 154962,
  155586, 156066, 156546, 156988, 157323, 157774, 158213, 158622, 159067, 159464, 159932, 160440, 161033, 161574, 162069, 162526,
  162975, 163450, 163982, 164679, 165353, 166023, 166733, 167489, 168295, 169016, 169533, 170017, 170481, 170942, 171340, 171767,
  172244, 172656, 173138, 173613, 173989, 174422, 174908, 175447, 175885, 176331, 176722, 177128, 177616, 178103, 178871, 180522,
  181353, 181763, 182271, 182856, 183506, 184085, 184857, 185392, 185849, 186411, 186868, 187551, 188243, 188762, 189248, 189764,
  190272, 190927, 191693, 192177, 192665, 193462, 194441, 195432, 196029, 196716, 197304, 198086, 198611, 199366, 200110, 200639,
  201185, 201676, 202387, 203305, 204238, 204899, 205423, 206038, 206880, 207781, 208688, 209533, 210109, 210826, 211871, 213210};
```

Calculate the differences between the average tap times and the corrected data, removing the extremities:

```
diffs1 = pid9048x06avg - pid9048x06corrected;
diffs1 = Drop[diffs1, 1]; (* drop the first pickup beat *)
diffs1 = Drop[diffs1, 3]; (* drop the first measure of the piece *)
diffs1 = Drop[diffs1, -3]; (* drop the last measure of the piece *)
diffs1 = N[diffs1 - Mean[diffs1]]; (* estimate correction for alignment error *)
beats1 = Transpose[Partition[diffs1, 3]];
```

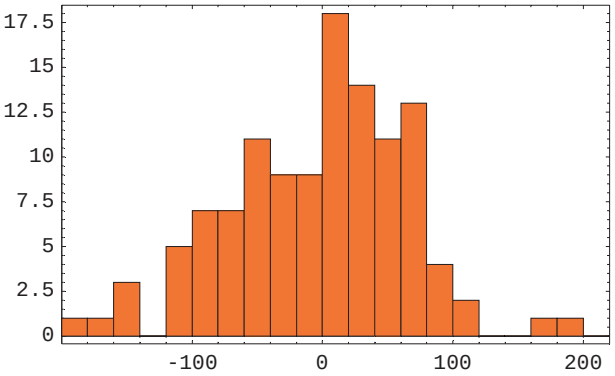
First display a histogram of all metrical beat displacements.

```
plot1 = Histogram[diffs1, Frame → True, HistogramRange → {-200, 200}, HistogramCategories → 20, BarStyle → {RGBColor[0.5, 0.5, 0.5]}];
```

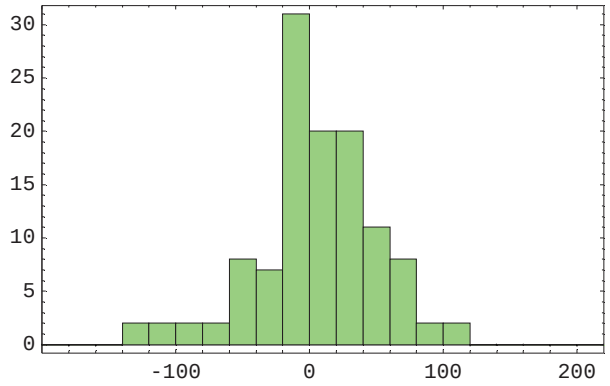


Now display separate histograms for individual metrical position in the meter:

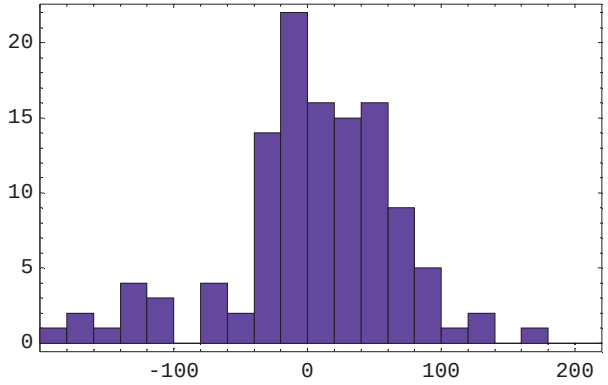
```
plot11 = Histogram[beats1[[1]], Frame → True, HistogramRange → {-200, 200}, HistogramCategories → 20, BarStyle → {RGBColor[1, 0, 0]}];
```



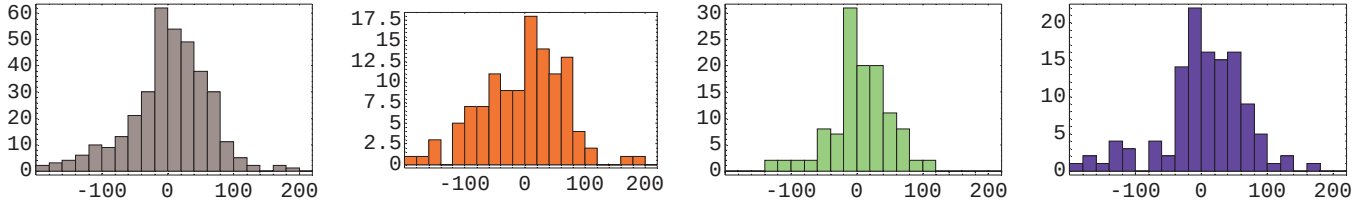

```
plot12 = Histogram[beats1[[2]], Frame → True, HistogramRange → {-200, 200}, HistogramCategories → 20, BarStyle → {RGBColor[0, 1, 0]}];
```



```
plot13 = Histogram[beats1[[3]], Frame → True, HistogramRange → {-200, 200}, HistogramCategories → 20, BarStyle → {RGBColor[0, 0, 1]}];
```




```
Show[GraphicsArray[{plot1, plot11, plot12, plot13}]];
```




```
{DispersionReport[diffs1]} // Transpose // TableForm
```

```
Variance → 3882.89
StandardDeviation → 62.3128
SampleRange → 450.
MeanDeviation → 46.4971
MedianDeviation → 35.5
QuartileDeviation → 34.5
```

```
stats1 = Map[DispersionReport, beats1] // N;
TableForm[Transpose[stats1]]
```

```
Variance → 5211.62                Variance → 2442.79                Variance → 4014.82
StandardDeviation → 72.1915       StandardDeviation → 49.4246       StandardDeviation → 63.3626
SampleRange → 450.                SampleRange → 335.                SampleRange → 359.
MeanDeviation → 56.7031           MeanDeviation → 36.               MeanDeviation → 47.0678
MedianDeviation → 48.5            MedianDeviation → 27.             MedianDeviation → 32.
QuartileDeviation → 49.           QuartileDeviation → 25.5          QuartileDeviation → 32.
```

Here is a table of the average deviation from the correct beat position, and the standard deviation for each beat:

```
info1 = Transpose[{Range[3], N[Map[Mean, Abs[beats1]]],
    Map[(StandardDeviation /. #) &, stats1]}] // TableForm
```

```
1      56.0959      72.1915
2      36.0801      49.4246
3      47.3151      63.3626
```

■ Mazurka in A minor, Op. 7, No. 2; Friedman 1930

```
pid5667230x09corrected = {1931, 2483, 2966, 3483, 3851, 4200, 4697, 4957, 5485, 5850, 6146, 6570, 6974, 7346, 7692, 8118, 8465, 8925, 9329, 96
   10130, 10533, 11109, 11765, 12183, 12677, 13220, 13750, 14088, 14438, 14892, 15058, 15531, 15911, 16217, 16639, 17207, 17630, 17939,
   18343, 18805, 19330, 19839, 20235, 20841, 21461, 22224, 22889, 23495, 23892, 24332, 24829, 25276, 25672, 26149, 26585, 26975, 27441,
   27902, 28285, 28833, 29693, 30364, 30819, 31315, 31705, 32195, 32706, 33090, 33620, 34136, 34654, 35438, 36100, 36782, 37254, 37592,
   37988, 38423, 38672, 39182, 39606, 39977, 40298, 40829, 41250, 41628, 42151, 42730, 43289, 43942, 44489, 45161, 45754, 46452, 47110,
   47701, 48107, 48557, 49084, 49524, 49900, 50339, 50738, 51104, 51544, 51960, 52305, 52755, 53422, 54055, 54514, 55018, 55454, 55917,
   56367, 56788, 57286, 57845, 58400, 59273, 60025, 60843, 61396, 61722, 62178, 62701, 62973, 63483, 63922, 64309, 64708, 65247, 65693,
   66043, 66474, 66941, 67623, 68365, 69201, 69931, 70547, 71097, 71549, 72190, 72940, 73547, 73964, 74355, 74752, 75117, 75424, 75738,
   76122, 76443, 76746, 77123, 77437, 77746, 78129, 78417, 78766, 79137, 79413, 79716, 80089, 80368, 80726, 81226, 81604, 82064, 82408,
   82714, 83061, 83434, 83745, 84101, 84478, 84738, 85106, 85607, 86014, 86390, 86745, 87057, 87398, 87752, 88124, 88728, 89208, 89603,
   90218, 90866, 91498, 92532, 92877, 93262, 93627, 94002, 94314, 94618, 95015, 95328, 95621, 95996, 96308, 96615, 97007, 97297, 97625,
   98014, 98292, 98604, 99049, 99358, 99673, 100264, 100789, 101259, 101677, 102039, 102352, 102721, 103091, 103443, 103792, 104056, 104412,
   104922, 105369, 105762, 106117, 106432, 106721, 107097, 107455, 108056, 108660, 109235, 110161, 111006, 112101, 112923, 113484, 113968,
   114368, 114733, 115094, 115399, 115818, 116071, 116357, 116746, 117092, 117428, 117785, 118084, 118397, 118735, 119084, 119389, 119840,
   120129, 120544, 121837, 122804, 123505, 124046, 124339, 124711, 125239, 125510, 126026, 126420, 126813, 127259, 127705, 128099, 128396,
   128932, 129328, 129803, 130224, 130604, 131047, 131461, 132029, 132648, 133429, 134285, 134863, 135392, 135684, 136017, 136545, 136719,
   137302, 137633, 137989, 138315, 138806, 139275, 139632, 140144, 140677, 141250, 141940, 142520, 143596, 144552, 145720, 146582, 148323};
```

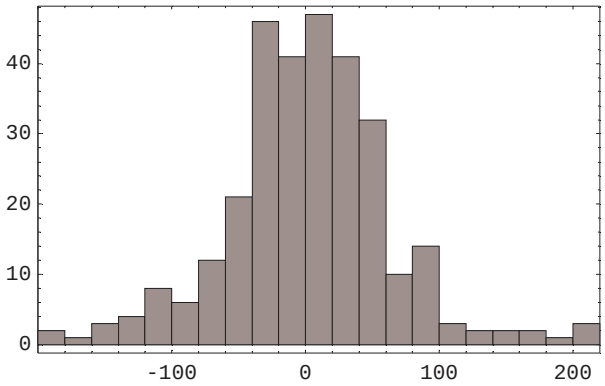
```
pid5667230x09avg = {2177, 2652, 3041, 3441, 3893, 4254, 4715, 5131, 5485, 5847, 6203, 6566, 7057, 7425, 7704, 8102, 8529, 8925, 9370, 9782, 101
    10582, 11104, 11773, 12424, 12780, 13241, 13671, 14154, 14498, 14926, 15299, 15614, 15913, 16238, 16613, 17116, 17652, 17970, 18332,
    18754, 19287, 19890, 20440, 20891, 21393, 22025, 22931, 23622, 24029, 24395, 24841, 25323, 25715, 26144, 26617, 27012, 27445, 27912,
    28354, 28800, 29576, 30248, 30855, 31346, 31763, 32184, 32678, 33155, 33601, 34142, 34662, 35503, 36100, 36811, 37237, 37670, 38051,
    38545, 38904, 39245, 39600, 39967, 40388, 40789, 41274, 41621, 42065, 42667, 43342, 43962, 44613, 45200, 45754, 46381, 47180, 47792,
    48239, 48627, 49084, 49590, 49987, 50427, 50830, 51180, 51591, 52016, 52400, 52819, 53445, 53932, 54533, 54990, 55454, 55917, 56408,
    56817, 57280, 57806, 58400, 59307, 60091, 60799, 61307, 61799, 62150, 62701, 63175, 63562, 63893, 64336, 64725, 65215, 65707, 66050,
    66456, 66926, 67497, 68260, 68938, 69883, 70483, 71087, 71597, 72114, 72779, 73608, 74003, 74385, 74802, 75171, 75505, 75816, 76139,
    76498, 76806, 77138, 77478, 77796, 78141, 78508, 78807, 79160, 79514, 79796, 80120, 80500, 80778, 81210, 81732, 82104, 82455, 82803,
    83092, 83458, 83813, 84175, 84519, 84867, 85125, 85660, 86064, 86444, 86812, 87141, 87445, 87800, 88198, 88788, 89287, 89708, 90285,
    90988, 91667, 92381, 93012, 93334, 93694, 94045, 94381, 94709, 95027, 95380, 95696, 96040, 96370, 96678, 97034, 97386, 97681, 98032,
    98382, 98669, 99034, 99446, 99795, 100276, 100826, 101280, 101662, 102054, 102379, 102733, 103110, 103491, 103832, 104162, 104436,
    104941, 105380, 105762, 106117, 106477, 106783, 107133, 107531, 108130, 108660, 109209, 110161, 111006, 111907, 112992, 113468, 113944,
    114421, 114792, 115124, 115443, 115796, 116163, 116460, 116778, 117096, 117428, 117791, 118164, 118469, 118811, 119134, 119440, 119851,
    120264, 120662, 122107, 122847, 123511, 123999, 124458, 124794, 125256, 125704, 126050, 126411, 126838, 127259, 127830, 128200, 128515,
    128866, 129379, 129803, 130251, 130685, 131140, 131532, 132029, 132726, 133356, 134065, 134922, 135375, 135797, 136140, 136559, 136957,
    137302, 137652, 138030, 138424, 138839, 139285, 139670, 140074, 140611, 141250, 141940, 142705, 143579, 144503, 145677, 146830, 148323};
```

Calculate the differences between the average tap times and the corrected data, removing the extremities:

```
diffs2 = pid5667230x09avg - pid5667230x09corrected;
diffs2 = Drop[diffs2, 1]; (* drop the first pickup beat *)
diffs2 = Drop[diffs2, 3]; (* drop the first measure of the piece *)
diffs2 = Drop[diffs2, -3]; (* drop the last measure of the piece *)
diffs2 = N[diffs2 - Mean[diffs2]]; (* estimate correction for alignment error *)
beats2 = Transpose[Partition[diffs2, 3]];
```

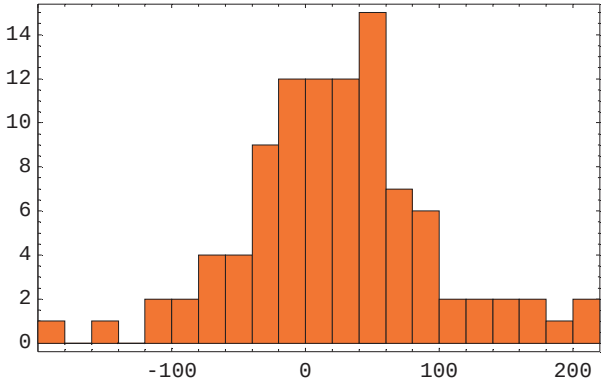
First display a histogram of all metrical beat displacements.

```
plot2 = Histogram[diffs2, Frame → True, HistogramRange → {-200, 200}, HistogramCategories → 20, BarStyle → {RGBColor[0.5, 0.5, 0.5]}];
```

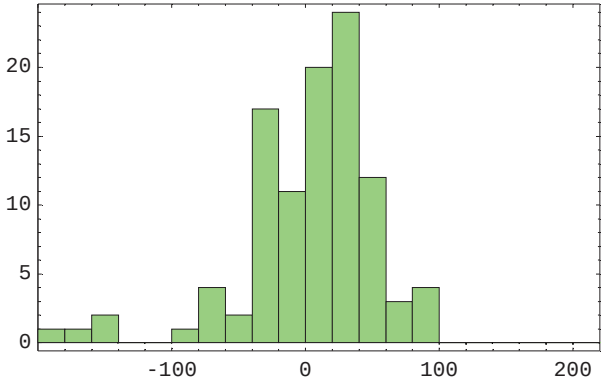


Now display separate histograms for individual metrical position in the meter:

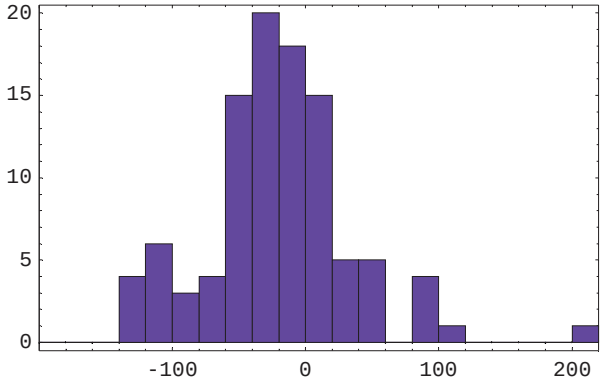
```
plot21 = Histogram[beats2[[1]], Frame → True, HistogramRange → {-200, 200}, HistogramCategories → 20, BarStyle → {RGBColor[1, 0, 0]}];
```



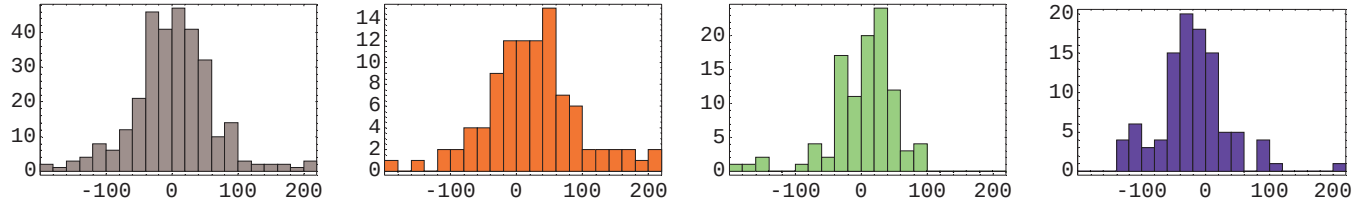

```
plot22 = Histogram[beats2[[2]], Frame → True, HistogramRange → {-200, 200}, HistogramCategories → 20, BarStyle → {RGBColor[0, 1, 0]}];
```




```
plot23 = Histogram[beats2[[3]], Frame → True, HistogramRange → {-200, 200}, HistogramCategories → 20, BarStyle → {RGBColor[0, 0, 1]}];
```



```
Show[GraphicsArray[{plot2, plot21, plot22, plot23}]];
```




```
{DispersionReport[diffs2]} // Transpose // TableForm
```

```
Variance → 4806.57
StandardDeviation → 69.3294
SampleRange → 533.
MeanDeviation → 49.4014
MedianDeviation → 37.5
QuartileDeviation → 35.
```

```
stats2 = Map[DispersionReport, beats2] // N;
TableForm[Transpose[stats2]]
```

```
Variance → 7901.5              Variance → 2524.86             Variance → 3578.66
StandardDeviation → 88.8904    StandardDeviation → 50.248     StandardDeviation → 59.8219
SampleRange → 504.             SampleRange → 274.             SampleRange → 375.
MeanDeviation → 62.3326        MeanDeviation → 37.1326        MeanDeviation → 41.5977
MedianDeviation → 39.          MedianDeviation → 26.          MedianDeviation → 28.5
QuartileDeviation → 40.5       QuartileDeviation → 30.5       QuartileDeviation → 29.
```

Here is a table of the average deviation from the correct beat position, and the standard deviation for each beat:

```
info2 = Transpose[{Range[3], N[Map[Mean, Abs[beats2]]],
    Map[(StandardDeviation /. #) &, stats2]}] // TableForm
```

```
1      64.6236      88.8904
2      37.8883      50.248
3      45.6924      59.8219
```

■ Mazurka in F minor, Op. 7, No. 3; Friedman 1930

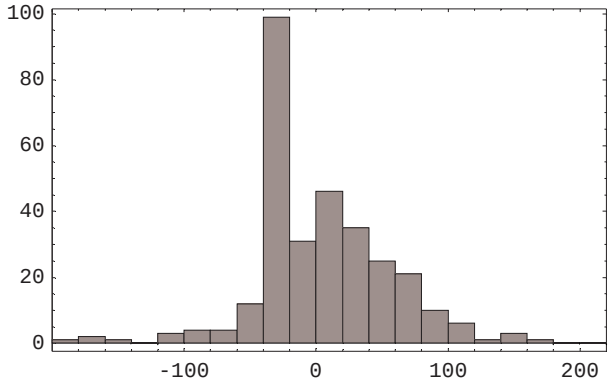
```
pid5667230x10corrected = {2262, 2859, 3250, 3559, 3897, 4205, 4575, 4941, 5285, 5582, 5935, 6297, 6664, 7020, 7306, 7579, 7956, 8318, 8615, 88
   9242, 9624, 9997, 10386, 11103, 11860, 12188, 12483, 12736, 13118, 13378, 13678, 13961, 14296, 14566, 14843, 15200, 15489, 15724,
   16025, 16245, 16583, 16930, 17280, 17505, 17804, 18154, 18512, 19505, 19960, 20270, 20516, 20783, 21181, 21486, 21763, 22025, 22360,
   22568, 22921, 23268, 23509, 23792, 24080, 24331, 24675, 24932, 25186, 25462, 25697, 26146, 26427, 26883, 27268, 27625, 28049, 28265,
   28604, 28884, 29170, 29428, 29704, 29978, 30217, 30527, 30872, 31187, 31397, 31619, 31959, 32207, 32526, 32801, 33026, 33258, 33613,
   33857, 34084, 34466, 34703, 34979, 35307, 35545, 35766, 36123, 36286, 36597, 36839, 37152, 37526, 37937, 38181, 38412, 38798, 39043,
   39252, 39609, 39855, 40062, 40409, 40705, 41036, 41363, 41693, 42067, 42441, 42759, 43049, 43399, 43728, 43994, 44307, 44597, 44912,
   45248, 45533, 45864, 46246, 46569, 46906, 47272, 47543, 47824, 48192, 48491, 48825, 49133, 49439, 49806, 50164, 50463, 50778, 51133,
   51456, 51698, 52042, 52333, 52659, 52989, 53267, 53607, 53965, 54222, 54607, 55039, 55422, 55880, 56337, 57093, 57662, 58319, 58605,
   58839, 59358, 59669, 60007, 60345, 60682, 60992, 61387, 61742, 62047, 62382, 62634, 62876, 63252, 63526, 63877, 64212, 64573,
   64951, 65227, 65639, 66011, 66434, 66698, 66941, 67283, 67574, 67930, 68281, 68614, 68909, 69320, 69672, 70018, 70396, 70615,
   71006, 71411, 71778, 72170, 72533, 73005, 73452, 73862, 74473, 74995, 75833, 76442, 77007, 77448, 77908, 78369, 78724, 79113,
   79551, 79943, 80254, 80678, 80995, 81285, 81686, 82030, 82292, 82831, 83111, 83335, 83652, 83981, 84308, 84638, 84947, 85172,
   85569, 85836, 86195, 86526, 86855, 87190, 87511, 87907, 88599, 89405, 89741, 90029, 90270, 90636, 90923, 91187, 91486, 91830,
   92027, 92408, 92743, 92988, 93283, 93573, 93792, 94117, 94441, 94737, 95033, 95309, 95697, 96134, 97342, 97956, 98260, 98513,
   98779, 99176, 99484, 99691, 99998, 100315, 100511, 100915, 101208, 101446, 101723, 101990, 102215, 102569, 102861, 103170, 103408,
   103681, 103874, 104250, 104585, 104847, 105094, 105396, 105581, 105895, 106374, 106539, 106711, 106984, 107473, 107918, 108605};

pid5667230x10avg = {2465, 2852, 3243, 3604, 3921, 4261, 4569, 4935, 5279, 5604, 5928, 6291, 6658, 7014, 7343, 7654, 7950, 8312, 8608, 8913, 923
   9618, 9990, 10379, 11096, 11670, 12182, 12510, 12830, 13112, 13468, 13748, 14022, 14290, 14559, 14837, 15194, 15483, 15750, 16051,
   16332, 16610, 16924, 17209, 17542, 17831, 18148, 18589, 19499, 19909, 20294, 20599, 20873, 21159, 21509, 21790, 22062, 22354, 22637,
   22915, 23262, 23539, 23823, 24112, 24377, 24681, 25001, 25352, 25629, 25875, 26109, 26531, 26896, 27262, 27619, 28042, 28314, 28598,
   28893, 29164, 29422, 29723, 29985, 30262, 30520, 30888, 31181, 31461, 31709, 31953, 32259, 32519, 32795, 33096, 33341, 33607, 33939,
   34200, 34459, 34754, 34998, 35301, 35578, 35860, 36122, 36388, 36646, 36896, 37146, 37520, 37930, 38248, 38490, 38755, 39082, 39359,
   39627, 39908, 40162, 40416, 40750, 41030, 41357, 41733, 42060, 42435, 42797, 43093, 43411, 43721, 44003, 44320, 44619, 44922, 45230,
   45549, 45888, 46240, 46563, 46915, 47245, 47589, 47890, 48221, 48533, 48838, 49157, 49460, 49800, 50158, 50480, 50808, 51127, 51449,
   51772, 52073, 52369, 52675, 52999, 53303, 53635, 53972, 54280, 54601, 54971, 55415, 55874, 56434, 57015, 57602, 58156, 58724, 59033,
   59352, 59734, 60050, 60397, 60694, 61022, 61381, 61736, 62085, 62415, 62697, 62994, 63284, 63566, 63842, 64190, 64567, 64922, 65304,
   65615, 65975, 66353, 66729, 67027, 67332, 67631, 67924, 68275, 68608, 68966, 69313, 69666, 70012, 70355, 70669, 71024, 71405, 71772,
   72126, 72526, 72998, 73446, 73856, 74483, 74988, 75586, 76436, 76987, 77602, 78018, 78362, 78787, 79252, 79544, 79937, 80310,
   80672, 81058, 81399, 81680, 82024, 82384, 82681, 83047, 83369, 83677, 83975, 84302, 84632, 84941, 85227, 85536, 85912, 86189,
   86520, 86849, 87207, 87546, 87975, 88680, 89195, 89735, 90082, 90369, 90668, 90976, 91258, 91541, 91823, 92116, 92382, 92745,
   93029, 93332, 93619, 93901, 94183, 94475, 94770, 95066, 95370, 95691, 96225, 97336, 97861, 98287, 98623, 98921, 99192, 99508,
   99779, 100046, 100361, 100635, 100920, 101242, 101507, 101804, 102084, 102342, 102626, 102937, 103215, 103502, 103766, 104007,
   104262, 104579, 104840, 105134, 105415, 105665, 105960, 106251, 106533, 106838, 107100, 107326, 107622, 107959, 108280, 108736};
```

Calculate the differences between the average tap times and the corrected data, removing the extremities:

```
diffs3 = Drop[pid5667230x10avg, -2] - pid5667230x10corrected;
diffs3 = Drop[diffs3, 3]; (* drop the first measure of the piece *)
diffs3 = Drop[diffs3, -1]; (* drop the last measure of the piece *)
diffs3 = N[diffs3 - Mean[diffs3]]; (* estimate correction for alignment error *)
beats3 = Transpose[Partition[diffs3, 3]];
```

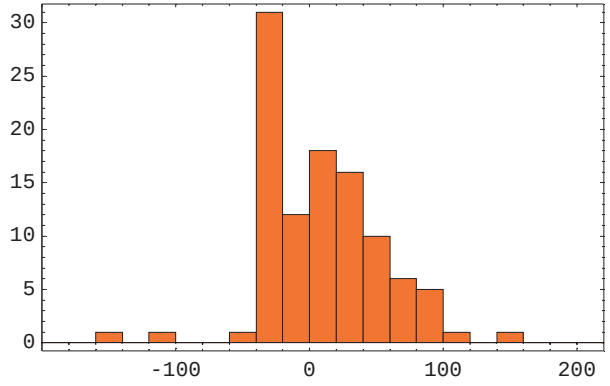
First display a histogram of all metrical beat displacements.

```
plot3 = Histogram[diffs3, Frame → True, HistogramRange → {-200, 200}, HistogramCategories → 20, BarStyle → {RGBColor[0.5, 0.5, 0.5]}];
```

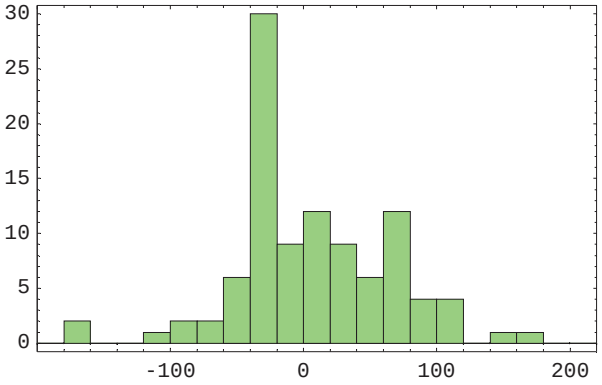


Now display separate histograms for individual metrical position in the meter:

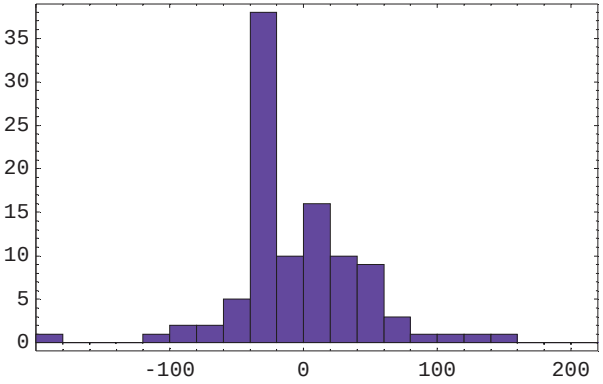
```
plot31 = Histogram[beats3[[1]], Frame → True, HistogramRange → {-200, 200}, HistogramCategories → 20, BarStyle → {RGBColor[1, 0, 0]}];
```



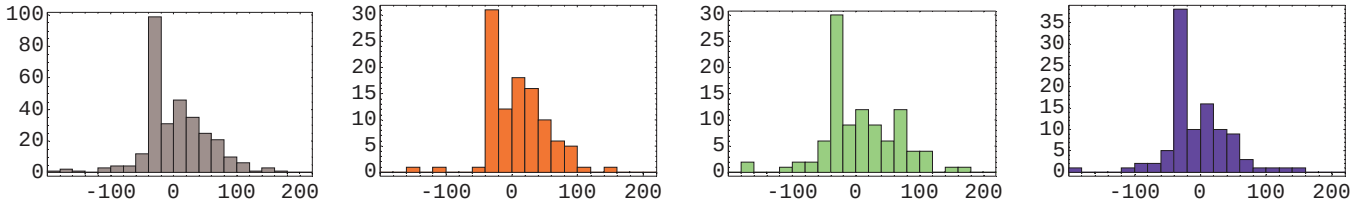
```
plot32 = Histogram[beats3[[2]], Frame → True, HistogramRange → {-200, 200}, HistogramCategories → 20, BarStyle → {RGBColor[0, 1, 0]}];
```




```
plot33 = Histogram[beats3[[3]], Frame → True, HistogramRange → {-200, 200}, HistogramCategories → 20, BarStyle → {RGBColor[0, 0, 1]}];
```




```
Show[GraphicsArray[{plot3, plot31, plot32, plot33}]];
```




```
{DispersionReport[diffs3]} // Transpose // TableForm
```

```
Variance → 3396.01
StandardDeviation → 58.2753
SampleRange → 490.
MeanDeviation → 41.2585
MedianDeviation → 25.
QuartileDeviation → 30.625
```

```
stats3 = Map[DispersionReport, beats3] // N;
TableForm[Transpose[stats3]]
```

Variance → 1939.28	Variance → 4393.87	Variance → 3711.94
StandardDeviation → 44.0373	StandardDeviation → 66.2863	StandardDeviation → 60.9257
SampleRange → 301.	SampleRange → 404.	SampleRange → 463.
MeanDeviation → 34.017	MeanDeviation → 48.9811	MeanDeviation → 38.7563
MedianDeviation → 37.	MedianDeviation → 27.	MedianDeviation → 22.
QuartileDeviation → 31.375	QuartileDeviation → 37.875	QuartileDeviation → 24.875

Here is a table of the average deviation from the correct beat position, and the standard deviation for each beat:

```
info3 = Transpose[{Range[3], N[Map[Mean, Abs[beats3]]],
    Map[(StandardDeviation /. #) &, stats3]}] // TableForm
```

1	34.3431	44.0373
2	48.9614	66.2863
3	40.4709	60.9257

■ Mazurka in F minor, Op. 7, No. 3; Rosen 1989

```
pid52932x05corrected = {707, 1194, 1612, 2116, 2634, 3045, 3425, 3817, 4236, 4663, 5078, 5510, 5885, 6249, 6661, 7106, 7570, 7916, 8442, 8784,
   9897, 10439, 11145, 12060, 12659, 13010, 13298, 13575, 14013, 14434, 14822, 15278, 15663, 15944, 16358, 16833, 17339, 17680, 18032,
   18371, 18809, 19254, 19754, 20135, 20527, 21116, 21629, 22610, 23099, 23639, 24176, 24488, 25146, 25574, 25994, 26702, 27070, 27298,
   27825, 28222, 28666, 29081, 29397, 29628, 30092, 30543, 31036, 31391, 31777, 32357, 33038, 33612, 34016, 34550, 34900, 35209, 35560,
   35933, 36247, 36693, 36981, 37376, 37627, 38106, 38612, 38919, 39193, 39451, 39757, 40065, 40436, 40791, 41075, 41415, 41967, 42586,
   43027, 43534, 43851, 44165, 44475, 44877, 45229, 45653, 45941, 46278, 46577, 46913, 47256, 47606, 47850, 48050, 48415, 48675, 49134,
   49427, 49703, 49968, 50229, 50646, 51050, 51390, 51707, 52139, 52480, 52809, 53163, 53501, 53797, 54130, 54439, 54758, 55077, 55395,
   55707, 56063, 56464, 56899, 57268, 57668, 58028, 58453, 58821, 59130, 59496, 59847, 60191, 60577, 60975, 61320, 61643, 62015, 62319,
   62628, 62966, 63269, 63595, 63925, 64217, 64882, 65368, 65753, 66151, 66599, 67012, 67501, 68133, 68635, 69258, 69828, 70163, 70475,
   70933, 71178, 71533, 71861, 72331, 72828, 73418, 73970, 74481, 74841, 75159, 75490, 75854, 76186, 76537, 76929, 77458, 77871, 78335,
   78873, 79412, 79763, 80080, 80379, 80775, 81090, 81413, 81776, 82249, 82702, 83270, 83776, 84351, 84712, 85057, 85364, 85777, 86142,
   86529, 86940, 87479, 87856, 88469, 89064, 89555, 90034, 90542, 91135, 91598, 92133, 92685, 93129, 93607, 94153, 94704, 95550, 96051,
   96513, 96885, 97305, 97843, 98311, 98705, 99140, 99580, 100036, 100523, 100942, 101360, 101739, 102078, 102625, 103022, 103557, 104001,
   104496, 105131, 105753, 106577, 107665, 108289, 108604, 108941, 109191, 109594, 110015, 110425, 110790, 111142, 111349, 111750, 112114,
   112564, 112871, 113113, 113346, 113722, 114148, 114639, 114988, 115328, 116058, 116598, 117980, 118716, 119213, 119641, 119924, 120382,
   121028, 121488, 121967, 122279, 122543, 123080, 123543, 123957, 124322, 124662, 124896, 125466, 125978, 126443, 126996, 127682,
   127942, 128444, 128949, 129432, 130024, 130618, 130904, 131396, 132127, 132741, 133554, 134702, 135141, 135919, 136992, 137993};
```

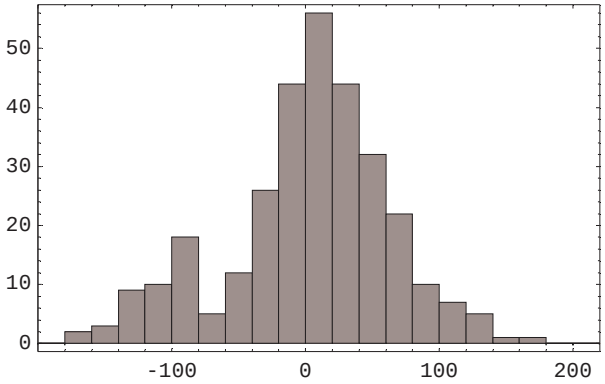
```
pid52932x05avg = {911, 1218, 1612, 2023, 2541, 3030, 3463, 3796, 4193, 4663, 5060, 5485, 5911, 6230, 6661, 7047, 7551, 7942, 8353, 8799, 9297,
  10428, 11164, 12066, 12646, 13018, 13379, 13653, 14074, 14479, 14832, 15159, 15675, 15963, 16409, 16811, 17298, 17660, 18011, 18306,
  18814, 19301, 19749, 20135, 20537, 21128, 21734, 22656, 23163, 23499, 24166, 24480, 24971, 25666, 26043, 26406, 27139, 27454, 27899,
  28337, 28702, 29054, 29512, 29731, 30143, 30596, 31036, 31402, 31806, 32308, 32923, 33589, 33984, 34461, 34916, 35209, 35560, 35890,
  36268, 36608, 36965, 37313, 37659, 37958, 38517, 38879, 39193, 39492, 39767, 40080, 40420, 40772, 41046, 41415, 41854, 42527, 42986,
  43404, 43871, 44159, 44489, 44784, 45184, 45551, 45920, 46253, 46577, 46899, 47314, 47606, 47891, 48159, 48415, 48720, 49028, 49413,
  49703, 50013, 50342, 50695, 51013, 51390, 51748, 52105, 52480, 52832, 53163, 53501, 53814, 54115, 54455, 54770, 55092, 55418, 55742,
  56111, 56464, 56811, 57268, 57629, 58016, 58402, 58870, 59209, 59496, 59838, 60182, 60577, 60975, 61320, 61681, 62003, 62339, 62660,
  62989, 63304, 63625, 63973, 64293, 64747, 65368, 65817, 66179, 66550, 67012, 67494, 68036, 68642, 69121, 69688, 70181, 70508, 70902,
  71259, 71561, 71898, 72268, 72744, 73262, 73937, 74432, 74878, 75187, 75497, 75859, 76186, 76537, 76920, 77363, 77849, 78256, 78785,
  79288, 79794, 80113, 80409, 80775, 81116, 81455, 81796, 82202, 82665, 83144, 83751, 84227, 84750, 85083, 85409, 85745, 86105, 86529,
  86933, 87391, 87841, 88307, 88925, 89523, 90054, 90559, 91025, 91598, 92026, 92589, 93153, 93581, 94050, 94633, 95567, 96037, 96513,
  96953, 97337, 97727, 98268, 98683, 99140, 99566, 100002, 100435, 100942, 101322, 101798, 102201, 102582, 103033, 103535, 103961,
  104436, 105005, 105741, 106604, 107650, 108312, 108658, 108967, 109247, 109665, 110071, 110425, 110777, 111142, 111413, 111845, 112182,
  112530, 112874, 113196, 113392, 113790, 114167, 114610, 115004, 115323, 115881, 116598, 117989, 118620, 119125, 119776, 120002, 120446,
  120884, 121560, 121956, 122413, 122628, 123080, 123577, 124006, 124384, 124710, 125003, 125466, 125956, 126516, 126891, 127697, 128009,
  128504, 129000, 129476, 129948, 130749, 130994, 131466, 132021, 132732, 133337, 134721, 135246, 135986, 136823, 138080, 139413};
```

Calculate the differences between the average tap times and the corrected data, removing the extremities:

```
diffs4 = Drop[pid52932x05avg, -3] - Drop[pid52932x05corrected, -2];
diffs4 = Drop[diffs4, 3]; (* drop the first measure of the piece *)
diffs4 = N[diffs4 - Mean[diffs4]]; (* estimate correction for alignment error *)
beats4 = Transpose[Partition[diffs4, 3]];
```

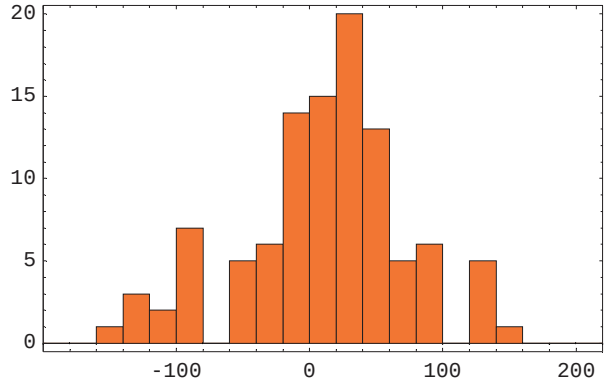
First display a histogram of all metrical beat displacements.

```
plot4 = Histogram[diffs4, Frame → True, HistogramRange → {-200, 200}, HistogramCategories → 20, BarStyle → {RGBColor[0.5, 0.5, 0.5]}];
```

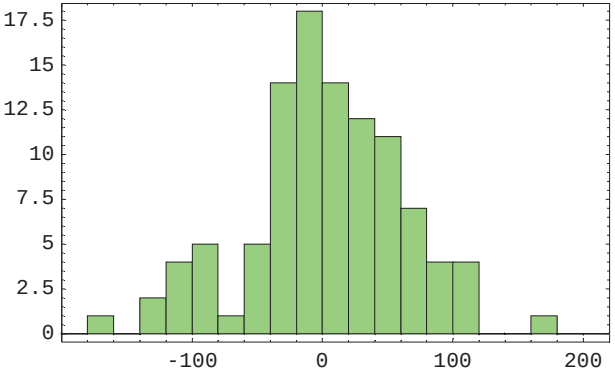


Now display separate histograms for individual metrical position in the meter:

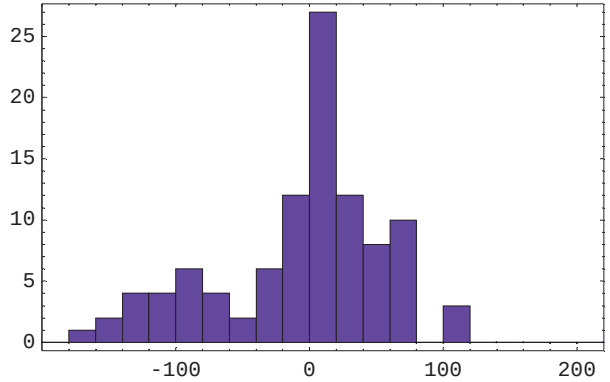
```
plot41 = Histogram[beats4[[1]], Frame → True, HistogramRange → {-200, 200}, HistogramCategories → 20, BarStyle → {RGBColor[1, 0, 0]}];
```



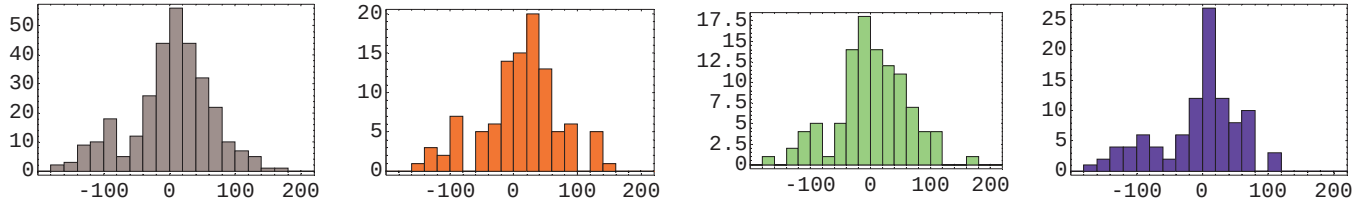

```
plot42 = Histogram[beats4[[2]], Frame → True, HistogramRange → {-200, 200}, HistogramCategories → 20, BarStyle → {RGBColor[0, 1, 0]}];
```




```
plot43 = Histogram[beats4[[3]], Frame → True, HistogramRange → {-200, 200}, HistogramCategories → 20, BarStyle → {RGBColor[0, 0, 1]}];
```



```
Show[GraphicsArray[{plot4, plot41, plot42, plot43}]];
```




```
{DispersionReport[diffs4]} // Transpose // TableForm
```

```
Variance → 4159.74
StandardDeviation → 64.496
SampleRange → 452.
MeanDeviation → 47.9149
MedianDeviation → 34.
QuartileDeviation → 34.5
```

```
stats4 = Map[DispersionReport, beats4] // N;
TableForm[Transpose[stats4]]
```

```
Variance → 3700.09              Variance → 3636.95              Variance → 4989.45
StandardDeviation → 60.8283     StandardDeviation → 60.3072     StandardDeviation → 70.636
SampleRange → 283.              SampleRange → 333.              SampleRange → 409.
MeanDeviation → 46.0929         MeanDeviation → 45.9959         MeanDeviation → 52.6919
MedianDeviation → 34.           MedianDeviation → 37.           MedianDeviation → 31.
QuartileDeviation → 31.875      QuartileDeviation → 36.875      QuartileDeviation → 33.
```

Here is a table of the average deviation from the correct beat position, and the standard deviation for each beat:

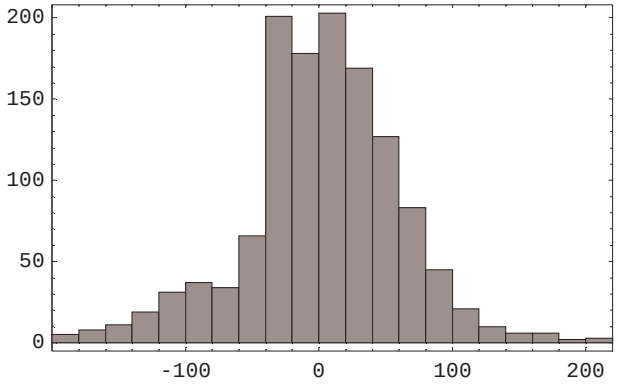
```
info4 = Transpose[{Range[3], N[Map[Mean, Abs[beats4]]],
    Map[(StandardDeviation /. #) &, stats4]}] // TableForm
```

```
1      47.7545      60.8283
2      46.0126      60.3072
3      49.9777      70.636
```

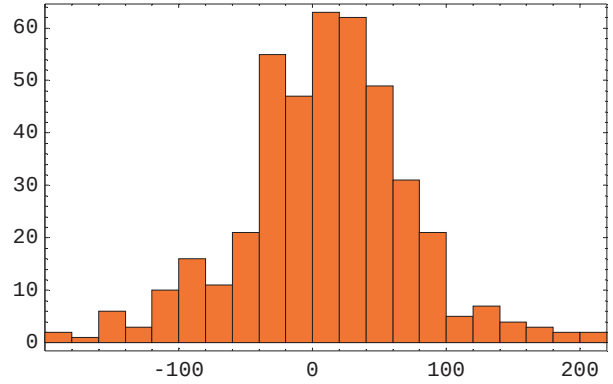
■ Accuracy summary for all beats of the four performances of two mazurkas:

```
alldiffs = Flatten[{diffs1, diffs2, diffs3, diffs4}];
allbeats = Transpose[Partition[alldiffs, 3]];
```

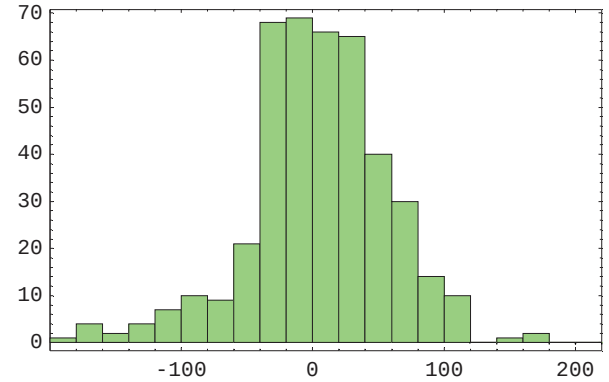
```
allplot = Histogram[alldiffs, Frame → True, HistogramRange → {-200, 200}, HistogramCategories → 20, BarStyle → {RGBColor[0.5, 0.5, 0.5]}];
```



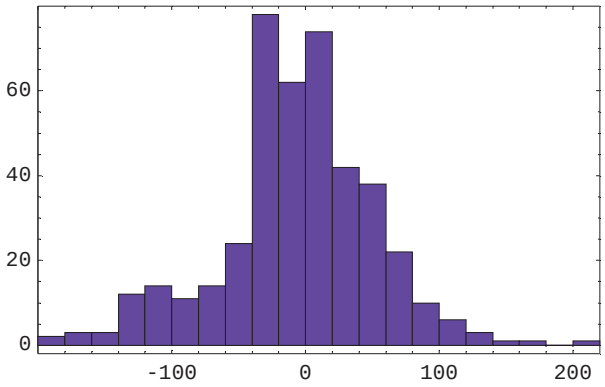

```
allplot1 = Histogram[allbeats[[1]], Frame → True, HistogramRange → {-200, 200}, HistogramCategories → 20, BarStyle → {RGBColor[1, 0, 0]}];
```



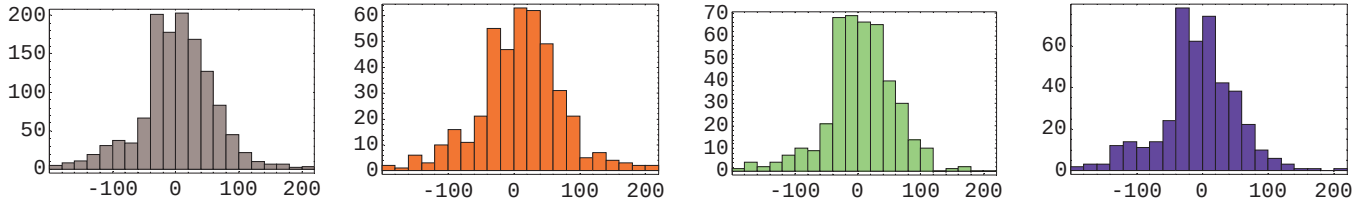

```
allplot2 = Histogram[allbeats[[2]], Frame → True, HistogramRange → {-200, 200}, HistogramCategories → 20, BarStyle → {RGBColor[0, 1, 0]}];
```



```
allplot3 = Histogram[allbeats[[3]], Frame → True, HistogramRange → {-200, 200}, HistogramCategories → 20, BarStyle → {RGBColor[0, 0, 1]}];
```




```
Show[GraphicsArray[{allplot, allplot1, allplot2, allplot3}]];
```




```
TableForm[Transpose[{DispersionReport[alldiffs]}]]
```

```
Variance → 4043.72
StandardDeviation → 63.5903
SampleRange → 554.089
MeanDeviation → 46.2687
MedianDeviation → 33.0631
QuartileDeviation → 33.0187
```

```
allstats = Map[DispersionReport, allbeats] // N;
TableForm[Transpose[allstats]]
```

```
Variance → 4724.76              Variance → 3200.94              Variance → 4105.01
StandardDeviation → 68.7369     StandardDeviation → 56.5768     StandardDeviation → 64.0704
SampleRange → 504.              SampleRange → 404.              SampleRange → 554.089
MeanDeviation → 50.0574         MeanDeviation → 41.9254         MeanDeviation → 45.7076
MedianDeviation → 40.           MedianDeviation → 33.3972       MedianDeviation → 29.
QuartileDeviation → 37.5444     QuartileDeviation → 32.5187     QuartileDeviation → 30.8341
```

Here is a table of the average deviation from the correct beat position, the absolute average deviation, and the standard deviation for each beat:

```
allinfo = Transpose[{Range[3], N[Map[Mean, allbeats]], N[Map[Mean, Abs[allbeats]]],
    Map[(StandardDeviation /. #) &, allstats]}] // TableForm
```

1	6.65571	50.8615	68.7369
2	1.80829	42.0291	56.5768
3	−8.46401	45.9155	64.0704

■ Tapping accuracy for individual trials

Examine the accuracy of beat tapping for Charles Rosen's 1989 peformance of Mazurka Op. 7, No. 3. Raw data is from `http://mazurka.org.uk/info/revcond/pid52932-05/pid52932-05-abs.ma`

```
SetDirectory["D:\mazurkas"];
<< pid52932-05-abs.ma

Length[pid52932x05abs]
```

```
20
```

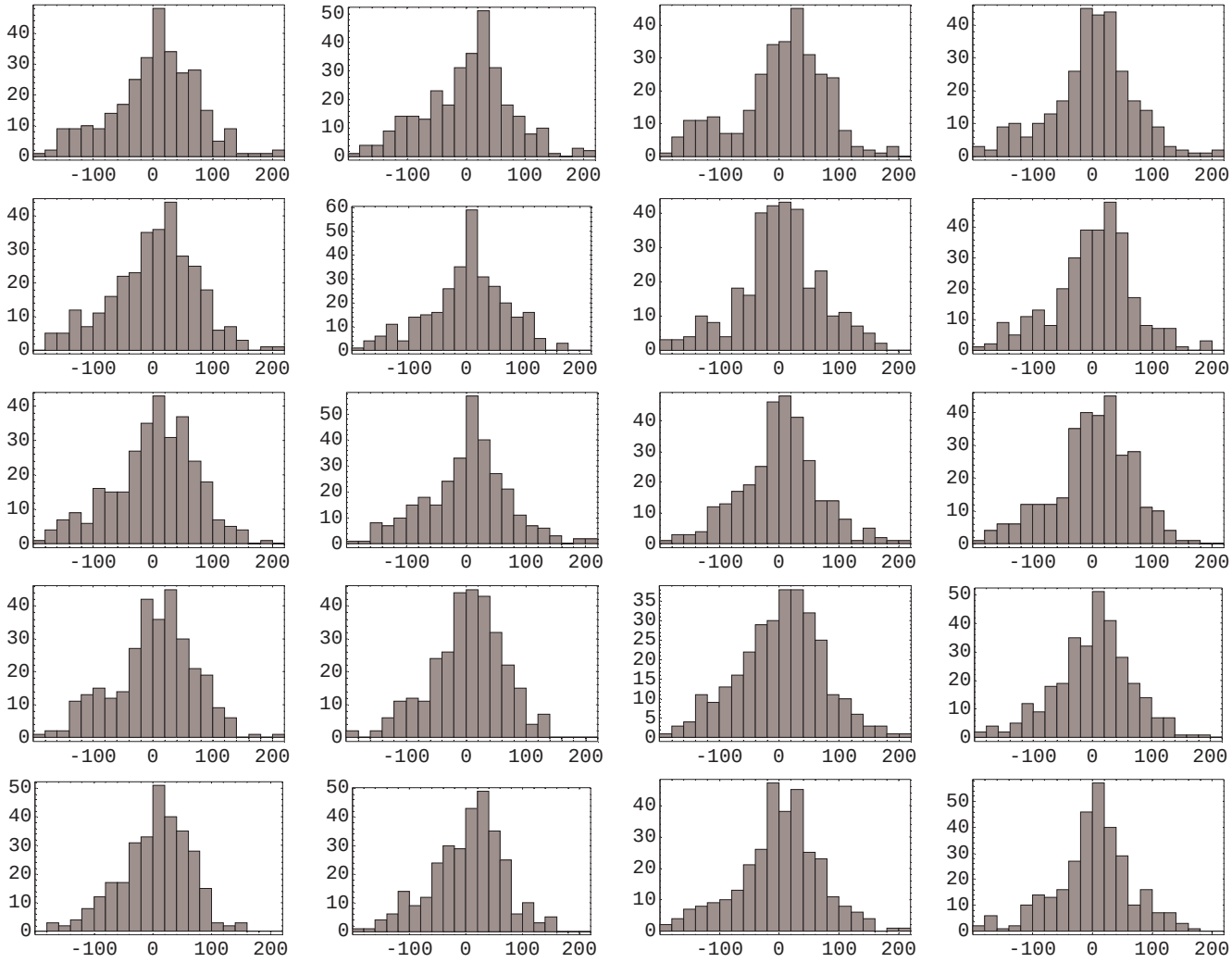
■ Mazurka in F minor, Op. 7, No. 3; Rosen 1989

Calculate the differences between the average tap times and the corrected data, removing the extremities:

```
rdifs = Map[(Drop[#, -3] - Drop[pid52932x05corrected, -2]) &, pid52932x05abs];
rdifs = Map[Drop[#, 3] &, rdifs]; (* drop the first measure of the piece *)
rdifs = Map[N[# - Mean[#]] &, rdifs]; (* estimate correction for alignment error *)
rbeats = Map[Transpose[Partition[#, 3]] &, rdifs];
```

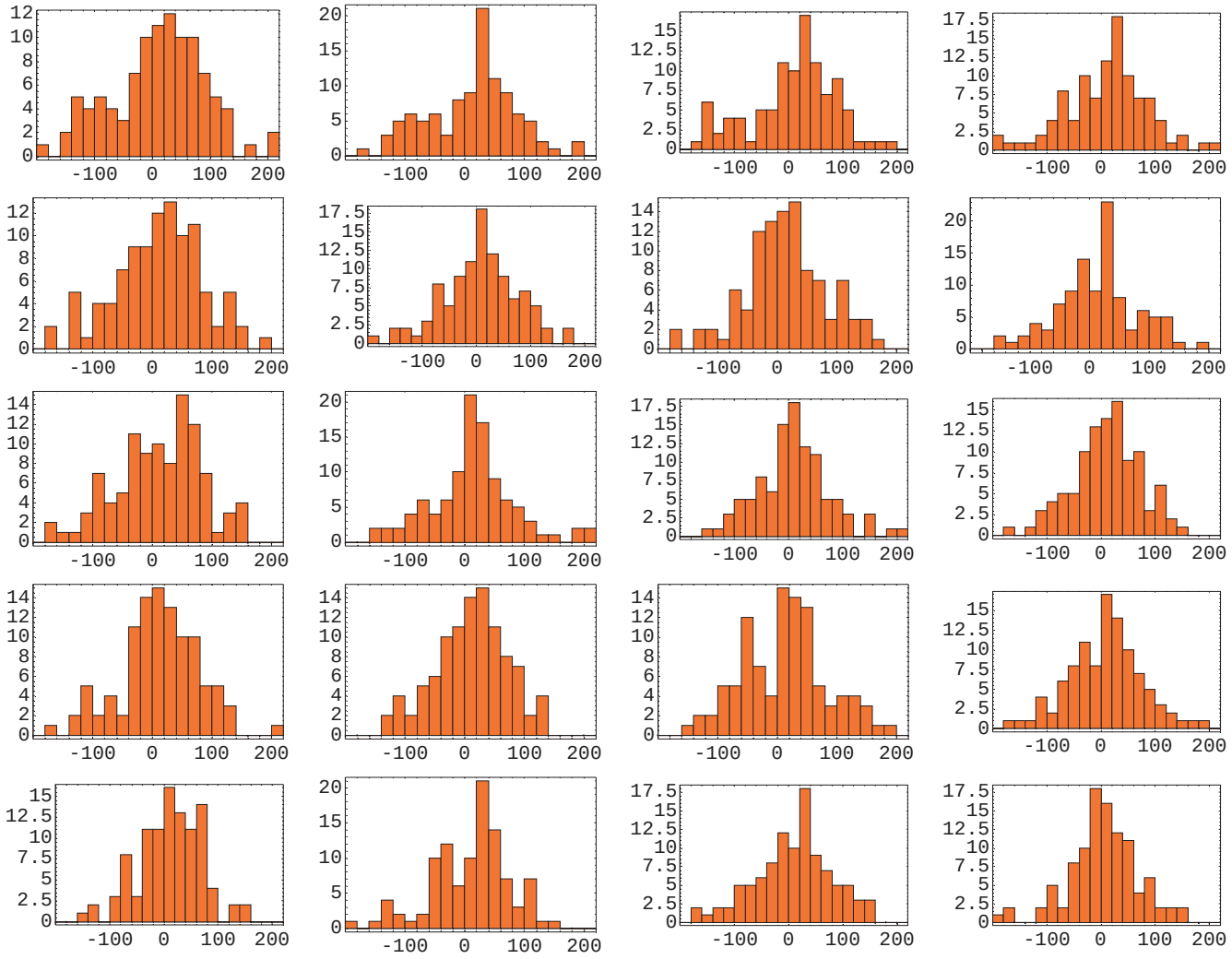
First display a histogram of all metrical beat displacements.

```
plots = Map[Histogram[#, Frame → True, HistogramRange → {-200, 200},
    HistogramCategories → 20, DisplayFunction → Identity, BarStyle → {RGBColor[0.5, 0.5, 0.5]}] &, rdifs];
Show[GraphicsArray[Partition[plots, 4]], DisplayFunction → $DisplayFunction];
```

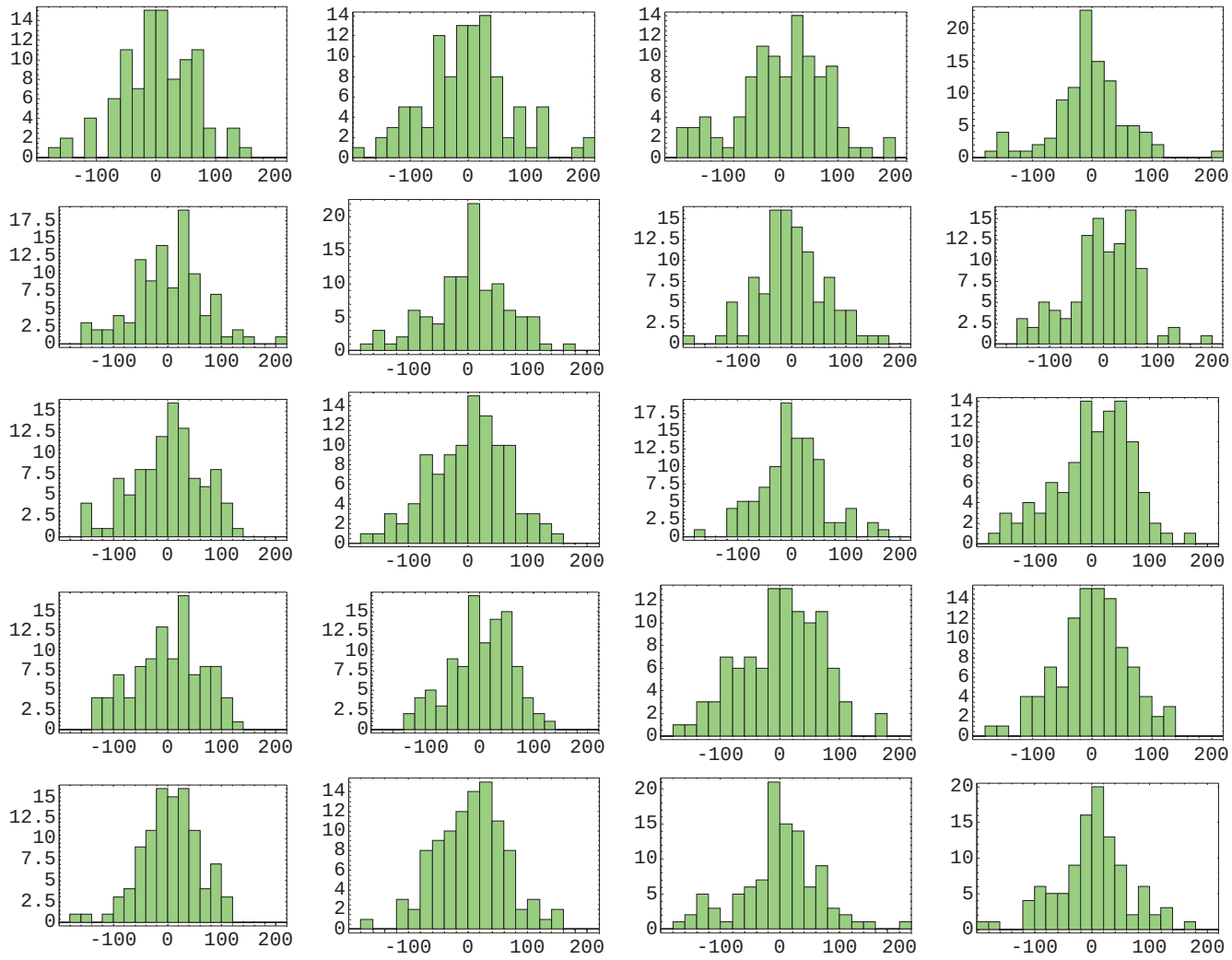


Now display separate histograms for individual metrical position in the meter, doing one beat at a time.

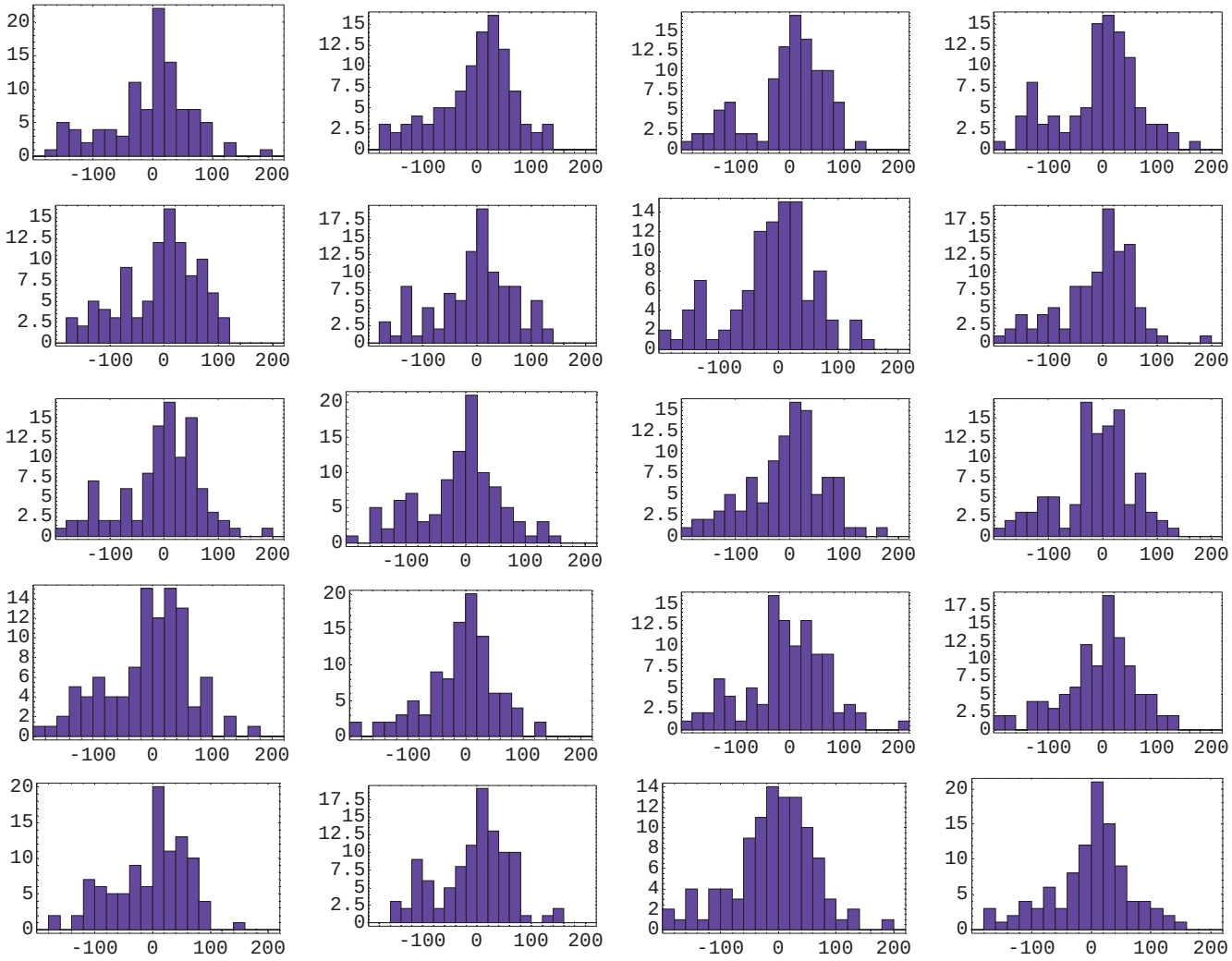
```
plots1 = Map[Histogram[#[[1]], Frame → True, HistogramRange → {-200, 200},
    HistogramCategories → 20, DisplayFunction → Identity, BarStyle → {RGBColor[1, 0, 0]}] &, rbeats];
Show[GraphicsArray[Partition[plots1, 4]], DisplayFunction → $DisplayFunction];
```



```
plots2 = Map[Histogram[#[[2]], Frame → True, HistogramRange → {-200, 200},
    HistogramCategories → 20, DisplayFunction → Identity, BarStyle → {RGBColor[0, 1, 0]}] &, rbeats];
Show[GraphicsArray[Partition[plots2, 4]], DisplayFunction → $DisplayFunction];
```



```
plots3 = Map[Histogram[#[[3]], Frame → True, HistogramRange → {-200, 200},
    HistogramCategories → 20, DisplayFunction → Identity, BarStyle → {RGBColor[0, 0, 1]}] &, rbeats];
Show[GraphicsArray[Partition[plots3, 4]], DisplayFunction → $DisplayFunction];
```



```
Map[{#[[1]], DispersionReport[ #[[2]] ]} &, Transpose[{Range[20], rdifs}]] // TableForm
```

	Variance → 6929.88 StandardDeviation → 83.2459
1	SampleRange → 571. MeanDeviation → 61.1577 MedianDeviation → 44. QuartileDeviation → 46.
2	Variance → 6213.44 StandardDeviation → 78.8254 SampleRange → 564. MeanDeviation → 60.3177 MedianDeviation → 45. QuartileDeviation → 45.125
3	Variance → 6318.33 StandardDeviation → 79.4879 SampleRange → 495. MeanDeviation → 60.643 MedianDeviation → 44. QuartileDeviation → 44.375
4	Variance → 6061.48 StandardDeviation → 77.8555 SampleRange → 577. MeanDeviation → 56.5196 MedianDeviation → 36. QuartileDeviation → 36.75
5	Variance → 5768.64 StandardDeviation → 75.9516 SampleRange → 525. MeanDeviation → 57.884 MedianDeviation → 45. QuartileDeviation → 44.875
6	Variance → 5112.25 StandardDeviation → 71.5 SampleRange → 459. MeanDeviation → 53.9907 MedianDeviation → 38. QuartileDeviation → 39.75
7	Variance → 5022.27 StandardDeviation → 70.868 SampleRange → 484. MeanDeviation → 53.395 MedianDeviation → 37. QuartileDeviation → 36.625
8	Variance → 4940.24 StandardDeviation → 70.2869 SampleRange → 511. MeanDeviation → 52.7888 MedianDeviation → 39. QuartileDeviation → 38.75
9	Variance → 5812.31 StandardDeviation → 76.2385 SampleRange → 607. MeanDeviation → 57.9524 MedianDeviation → 42. QuartileDeviation → 43.125

10	Variance → 5003.9 StandardDeviation → 70.7382 SampleRange → 509. MeanDeviation → 53.3253 MedianDeviation → 39. QuartileDeviation → 40.
11	Variance → 5031.98 StandardDeviation → 70.9365 SampleRange → 548. MeanDeviation → 51.8654 MedianDeviation → 38. QuartileDeviation → 37.875
12	Variance → 4646.28 StandardDeviation → 68.1636 SampleRange → 454. MeanDeviation → 52.4064 MedianDeviation → 38. QuartileDeviation → 37.375
13	Variance → 5111.12 StandardDeviation → 71.4921 SampleRange → 565. MeanDeviation → 54.792 MedianDeviation → 42. QuartileDeviation → 41.75
14	Variance → 4243.78 StandardDeviation → 65.1443 SampleRange → 446. MeanDeviation → 49.7277 MedianDeviation → 39. QuartileDeviation → 38.
15	Variance → 5554.76 StandardDeviation → 74.5302 SampleRange → 458. MeanDeviation → 57.9288 MedianDeviation → 44. QuartileDeviation → 47.
16	Variance → 4486.75 StandardDeviation → 66.9832 SampleRange → 463. MeanDeviation → 50.8673 MedianDeviation → 38. QuartileDeviation → 38.125
17	Variance → 4654.58 StandardDeviation → 68.2245 SampleRange → 482. MeanDeviation → 50.2288 MedianDeviation → 38. QuartileDeviation → 37.125
18	Variance → 4496.06 StandardDeviation → 67.0527 SampleRange → 396. MeanDeviation → 51.5992 MedianDeviation → 39. QuartileDeviation → 39.875
19	Variance → 4734.58 StandardDeviation → 68.8083 SampleRange → 399. MeanDeviation → 52.6682 MedianDeviation → 38. QuartileDeviation → 39.25

```
rstats = Map[DispersionReport, Flatten[rbeats, 1]] // N;
TableForm[Partition[rstats, 3]]
```

Variance → 6460.59 StandardDeviation → 80.3778 SampleRange → 401. MeanDeviation → 63.1409 MedianDeviation → 48. QuartileDeviation → 49.125	Variance → 6656.65 StandardDeviation → 81.5883 SampleRange → 486. MeanDeviation → 57.922 MedianDeviation → 47. QuartileDeviation → 45.875	Variance → 7470.48 StandardDeviation → 86.4319 SampleRange → 514. MeanDeviation → 63.8611 MedianDeviation → 37. QuartileDeviation → 41.
Variance → 5385.18 StandardDeviation → 73.3838 SampleRange → 360. MeanDeviation → 58.1318 MedianDeviation → 44. QuartileDeviation → 50.	Variance → 5861.65 StandardDeviation → 76.5614 SampleRange → 410. MeanDeviation → 57.9553 MedianDeviation → 42. QuartileDeviation → 44.	Variance → 7195.95 StandardDeviation → 84.829 SampleRange → 477. MeanDeviation → 64.2315 MedianDeviation → 39. QuartileDeviation → 48.625
Variance → 6835.67 StandardDeviation → 82.6781 SampleRange → 493. MeanDeviation → 62.4436 MedianDeviation → 46. QuartileDeviation → 43.875	Variance → 6166.36 StandardDeviation → 78.5261 SampleRange → 383. MeanDeviation → 61.4001 MedianDeviation → 48. QuartileDeviation → 47.875	Variance → 5956.73 StandardDeviation → 77.1799 SampleRange → 410. MeanDeviation → 58.5673 MedianDeviation → 38. QuartileDeviation → 38.75
Variance → 5499.17 StandardDeviation → 74.1564 SampleRange → 400. MeanDeviation → 56.7405 MedianDeviation → 42. QuartileDeviation → 43.125	Variance → 5635.45 StandardDeviation → 75.0696 SampleRange → 474. MeanDeviation → 50.7041 MedianDeviation → 35. QuartileDeviation → 30.25	Variance → 6901.57 StandardDeviation → 83.0757 SampleRange → 485. MeanDeviation → 62.2568 MedianDeviation → 38. QuartileDeviation → 46.25
Variance → 5839.72 StandardDeviation → 76.4181 SampleRange → 400. MeanDeviation → 59.5956 MedianDeviation → 50. QuartileDeviation → 48.625	Variance → 4670.7 StandardDeviation → 68.3425 SampleRange → 428. MeanDeviation → 52.1576 MedianDeviation → 41. QuartileDeviation → 40.375	Variance → 6548.7 StandardDeviation → 80.9241 SampleRange → 414. MeanDeviation → 62.6689 MedianDeviation → 50. QuartileDeviation → 54.5
Variance → 4624.13 StandardDeviation → 68.001 SampleRange → 359. MeanDeviation → 51.2412 MedianDeviation → 40. QuartileDeviation → 39.5	Variance → 4402.38 StandardDeviation → 66.3504 SampleRange → 342. MeanDeviation → 50.863 MedianDeviation → 38. QuartileDeviation → 38.75	Variance → 6239.65 StandardDeviation → 78.9915 SampleRange → 420. MeanDeviation → 60.2762 MedianDeviation → 45. QuartileDeviation → 43.625
Variance → 4710.56 StandardDeviation → 68.6335 SampleRange → 345. MeanDeviation → 52.2006 MedianDeviation → 41. QuartileDeviation → 41.375	Variance → 3955.2 StandardDeviation → 62.8904 SampleRange → 346. MeanDeviation → 49.219 MedianDeviation → 36. QuartileDeviation → 35.5	Variance → 6087.88 StandardDeviation → 78.0249 SampleRange → 472. MeanDeviation → 58.3631 MedianDeviation → 38. QuartileDeviation → 38.75
Variance → 4337.33 StandardDeviation → 65.8584 SampleRange → 355. MeanDeviation → 50.656 MedianDeviation → 39. QuartileDeviation → 35.25	Variance → 4498.46 StandardDeviation → 67.0706 SampleRange → 388. MeanDeviation → 50.5959 MedianDeviation → 42. QuartileDeviation → 42.5	Variance → 5663.01 StandardDeviation → 75.253 SampleRange → 461. MeanDeviation → 57.6243 MedianDeviation → 41. QuartileDeviation → 43.125
Variance → 5151.86 StandardDeviation → 71.7765 SampleRange → 325. MeanDeviation → 58.0034 MedianDeviation → 46. QuartileDeviation → 46.25	Variance → 5252.49 StandardDeviation → 72.4741 SampleRange → 490. MeanDeviation → 54.7925 MedianDeviation → 40. QuartileDeviation → 44.125	Variance → 6846.8 StandardDeviation → 82.7454 SampleRange → 502. MeanDeviation → 61.275 MedianDeviation → 42. QuartileDeviation → 42.625

Variance → 4834.71
StandardDeviation → 69.5321
SampleRange → 366.
MeanDeviation → 50.1333
MedianDeviation → 34.
QuartileDeviation → 34.25

Variance → 4249.87
StandardDeviation → 65.191
SampleRange → 348.
MeanDeviation → 49.09
MedianDeviation → 38.
QuartileDeviation → 37.875

Variance → 3910.15
StandardDeviation → 62.5312
SampleRange → 317.
MeanDeviation → 48.7528
MedianDeviation → 41.
QuartileDeviation → 40.375

Variance → 4380.15
StandardDeviation → 66.1827
SampleRange → 369.
MeanDeviation → 50.7814
MedianDeviation → 38.
QuartileDeviation → 38.375

Variance → 4507.61
StandardDeviation → 67.1388
SampleRange → 347.
MeanDeviation → 51.983
MedianDeviation → 39.
QuartileDeviation → 41.5

Variance → 5989.19
StandardDeviation → 77.3899
SampleRange → 427.
MeanDeviation → 59.4299
MedianDeviation → 47.
QuartileDeviation → 47.5

Variance → 4415.82
StandardDeviation → 66.4516
SampleRange → 348.
MeanDeviation → 50.6436
MedianDeviation → 41.
QuartileDeviation → 40.125

Variance → 4968.39
StandardDeviation → 70.4868
SampleRange → 482.
MeanDeviation → 50.9909
MedianDeviation → 42.
QuartileDeviation → 42.

Variance → 4057.87
StandardDeviation → 63.7014
SampleRange → 326.
MeanDeviation → 49.7221
MedianDeviation → 40.
QuartileDeviation → 39.625

Variance → 4888.13
StandardDeviation → 69.9151
SampleRange → 332.
MeanDeviation → 54.5771
MedianDeviation → 44.
QuartileDeviation → 42.375

Variance → 4197.98
StandardDeviation → 64.7918
SampleRange → 330.
MeanDeviation → 52.0477
MedianDeviation → 43.
QuartileDeviation → 44.375

Variance → 4535.51
StandardDeviation → 67.3462
SampleRange → 441.
MeanDeviation → 48.2438
MedianDeviation → 34.
QuartileDeviation → 34.875

Variance → 4626.36
StandardDeviation → 68.0174
SampleRange → 337.
MeanDeviation → 53.5841
MedianDeviation → 43.
QuartileDeviation → 43.125

Variance → 4369.61
StandardDeviation → 66.103
SampleRange → 268.
MeanDeviation → 54.1883
MedianDeviation → 49.
QuartileDeviation → 49.25

Variance → 3357.14
StandardDeviation → 57.9408
SampleRange → 272.
MeanDeviation → 46.6478
MedianDeviation → 41.
QuartileDeviation → 40.375

Variance → 4697.27
StandardDeviation → 68.5366
SampleRange → 337.
MeanDeviation → 54.9038
MedianDeviation → 50.
QuartileDeviation → 52.125

Variance → 3582.5
StandardDeviation → 59.854
SampleRange → 298.
MeanDeviation → 46.282
MedianDeviation → 34.
QuartileDeviation → 34.

Variance → 3507.17
StandardDeviation → 59.2214
SampleRange → 376.
MeanDeviation → 43.0764
MedianDeviation → 30.
QuartileDeviation → 30.5

Variance → 4440.29
StandardDeviation → 66.6355
SampleRange → 392.
MeanDeviation → 50.0967
MedianDeviation → 38.
QuartileDeviation → 40.

Variance → 4352.39
StandardDeviation → 65.9726
SampleRange → 365.
MeanDeviation → 49.5385
MedianDeviation → 31.
QuartileDeviation → 32.75

Variance → 5657.93
StandardDeviation → 75.2192
SampleRange → 444.
MeanDeviation → 56.8101
MedianDeviation → 39.
QuartileDeviation → 44.25

Variance → 6146.14
StandardDeviation → 78.3973
SampleRange → 476.
MeanDeviation → 59.0457
MedianDeviation → 40.
QuartileDeviation → 43.125

Variance → 5187.41
StandardDeviation → 72.0237
SampleRange → 415.
MeanDeviation → 53.7251
MedianDeviation → 34.
QuartileDeviation → 34.875

Variance → 6395.66
StandardDeviation → 79.9729
SampleRange → 532.
MeanDeviation → 59.7001
MedianDeviation → 43.
QuartileDeviation → 46.125

Variance → 4771.99
StandardDeviation → 69.0796
SampleRange → 442.
MeanDeviation → 49.9014
MedianDeviation → 31.
QuartileDeviation → 37.875

Variance → 5921.61
StandardDeviation → 76.952
SampleRange → 457.
MeanDeviation → 58.4615
MedianDeviation → 40.
QuartileDeviation → 39.25

Variance → 5404.45
StandardDeviation → 73.515
SampleRange → 419.
MeanDeviation → 55.5404
MedianDeviation → 36.
QuartileDeviation → 37.125

Variance → 5410.86
StandardDeviation → 73.5585
SampleRange → 446.
MeanDeviation → 57.2574
MedianDeviation → 41.
QuartileDeviation → 45.75

Variance → 4904.64
StandardDeviation → 70.0331
SampleRange → 396.
MeanDeviation → 54.5488
MedianDeviation → 41.
QuartileDeviation → 42.625

Variance → 4921.37
StandardDeviation → 70.1525
SampleRange → 390.
MeanDeviation → 53.084
MedianDeviation → 39.
QuartileDeviation → 39.5

Here is a table of the average deviation from the correct beat position, and the standard deviation for each beat:

```
ravg = Partition[Map[Mean, Flatten[rbeats, 1]], 3];
rerr = Partition[ Map[Mean, Abs[Flatten[rbeats, 1]]], 3];
rsd = Partition[Map[(StandardDeviation /. #) &, rstats], 3];
rrange = Flatten[Transpose[{Range[20] * 100 + Table[Range[3], {x, 1, 20}]}], 1 ];
Transpose[{rrange, ravg, rerr, rsd}] // TableForm
```

101	10.7735	64.5804	80.3778
102	3.56958	58.0198	81.5883
103	−14.343	60.8729	86.4319
201	12.9806	60.8513	73.3838
202	−0.854369	57.9636	76.5614
203	−12.1262	62.1381	84.829
301	6.59871	63.8275	82.6781
302	1.88026	61.5644	78.5261
303	−8.47896	56.537	77.1799
401	10.4401	58.2906	74.1564
402	1.88673	50.5123	75.0696
403	−12.3269	60.7558	83.0757
501	13.1392	60.8278	76.4181
502	0.411003	52.1696	68.3425
503	−13.5502	60.6546	80.9241
601	8.52427	52.3011	68.001
602	1.2233	51.0246	66.3504
603	−9.74757	58.6464	78.9915
701	14.301	53.5454	68.6335
702	−0.0485437	49.2213	62.8904
703	−14.2524	57.4182	78.0249
801	13.7767	52.2303	65.8584
802	1.17476	50.6301	67.0706
803	−14.9515	55.5059	75.253
901	11.4854	59.4555	71.7765
902	1.29126	54.8905	72.4741
903	−12.7767	59.5114	82.7454
1001	14.0291	52.839	69.5321
1002	0.728155	52.1255	64.7918
1003	−14.7573	55.0114	75.2192
1101	10.2362	49.7618	65.191
1102	2.0712	48.2236	67.3462
1103	−12.3074	57.6106	78.3973
1201	10.4337	50.3285	62.5312
1202	3.14239	53.8769	68.0174
1203	−13.5761	53.0139	72.0237
1301	11.6893	51.5502	66.1827
1302	0.495146	54.2124	66.103
1303	−12.1845	58.6133	79.9729
1401	7.07443	53.2835	67.1388
1402	3.56958	46.8903	57.9408
1403	−10.644	49.0093	69.0796
1501	8.19094	60.7605	77.3899
1502	1.51133	55.0194	68.5366
1503	−9.70227	58.0065	76.952

1601	7.30097	51.6505	66.4516
1602	2.00971	46.2816	59.854
1603	−9.31068	54.6699	73.515
1701	7.11327	51.9527	70.4868
1702	3.19094	43.314	59.2214
1703	−10.3042	55.4197	73.5585
1801	8.72168	51.33	63.7014
1802	1.01294	50.1852	66.6355
1803	−9.73463	53.2823	70.0331
1901	8.24919	55.5152	69.9151
1902	−0.168285	49.5369	65.9726
1903	−8.08091	52.9525	70.1525
2001	3.89644	46.2386	62.5385
2002	1.90615	46.5717	62.1844
2003	−5.80259	54.0078	75.3897

■ Checking the relation to pre-beat event durations

■ Macurka in A minor, Op. 7, No. 2

```
b2t1 = 2 / 88 // N
b3t1 = 65 / 88 // N
b3p5t1 = 17 / 88 // N
b3p75t1 = 4 / 88 // N
```

```
0.0227273
```

```
0.738636
```

```
0.193182
```

```
0.0454545
```

```
b2t1 + b3t1
```

```
0.761364
```

```
b1t2 = 28 / 88 // N
b1p5t2 = 15 / 88 // N
b1p67t2 = 25 / 88 // N
b1p75t2 = 20 / 88 // N
```

```
0.318182
```

```
0.170455
```

```
0.284091
```

```
0.227273
```

```
76 / 87 // N
```

```
0.873563
```

```
7 / 87 // N
```

```
0.0804598
```

```
(76 + 28 + 2 + 65) / (88 + 88 + 87) // N
```

```
0.65019
```

■ Mazurka in F minor, Op. 7, No 3

Beat 1:

```
10 / 104 // N
47 / 104 // N
42 / 104 // N
5 / 104 // N
```

```
0.0961538
```

```
0.451923
```

```
0.403846
```

```
0.0480769
```

Beat 2:

```
17 / 105 // N
27 / 105 // N
15 / 105 // N
46 / 105 // N
```

```
0.161905
```

```
0.257143
```

```
0.142857
```

```
0.438095
```

Beat 3:

```
60 / 95 // N
35 / 95 // N
```

```
0.631579
```

```
0.368421
```

```
(57 + 17 + 60) / (104 + 105 + 95) // N
```

```
0.440789
```

```
(57 + 17 + 60 + 76 + 28 + 2 + 65) / (104 + 105 + 95 + 88 + 88 + 87) // N
```

```
0.537919
```

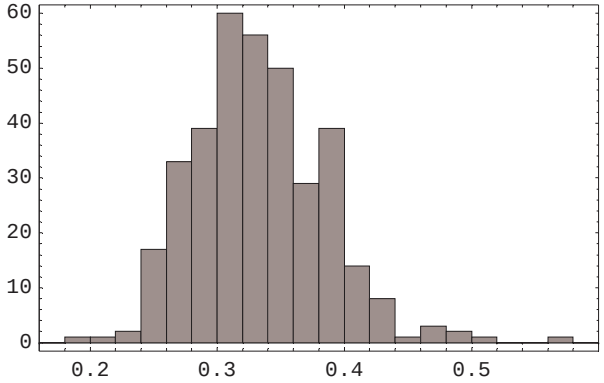
■ Beat durations for each beat

This section measures the average beat duration for each beat in the triple meter of the Mazurkas. Averaging for the "exact" performance by the pianists, and the average uncorrected reverse ducted taps of a listener. Durations are averaged over one measure to compensate for long-term tempo changes, but short-term tempo changes within a measures are more difficult to remove are ignored.

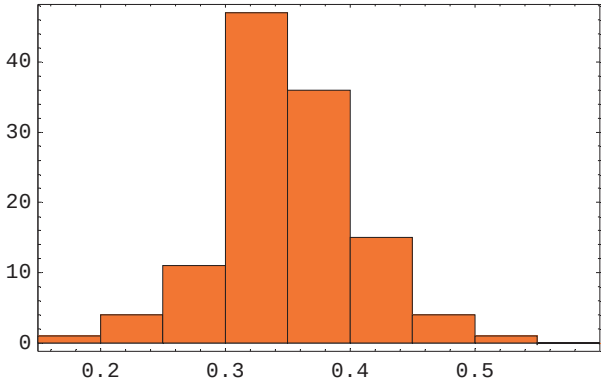
■ Mazurka in A minor, Op. 7, No. 2; Chiu 1999

■ corrected timing data

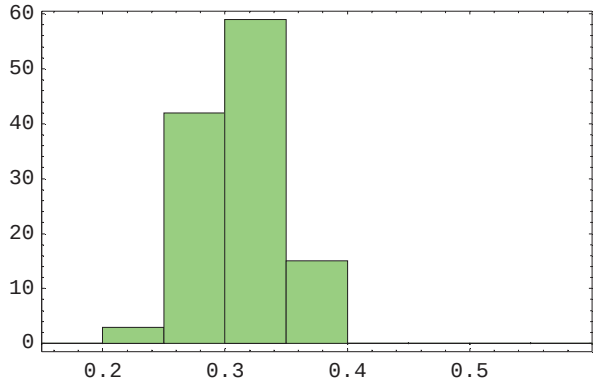
```
data1 = Drop[pid9048x06corrected, 1]; (* remove pickup note *)
beats1cor = Drop[RotateLeft[pid9048x06corrected, 1] - pid9048x06corrected, -1];
beats1cor = Drop[beats1cor, -3];(* remove last measure *)

measure1durs = Map[Apply[Plus, #] &, Partition[beats1cor, 3]];

newdata1 = Transpose[{measure1durs, Partition[beats1cor, 3]}];
fractions1 = Transpose[Map[{N[#[[2]][[1]] / #[[1]]],
        N[#[[2]][[2]] / #[[1]] ],
       N[#[[2]][[3]] / #[[1]] ]} &,
    newdata1]];

xplot1 = Histogram[Flatten[fractions1], Frame → True, HistogramRange → {0.16, 0.58}, BarStyle → {RGBColor[0.5, 0.5, 0.5]}];
```



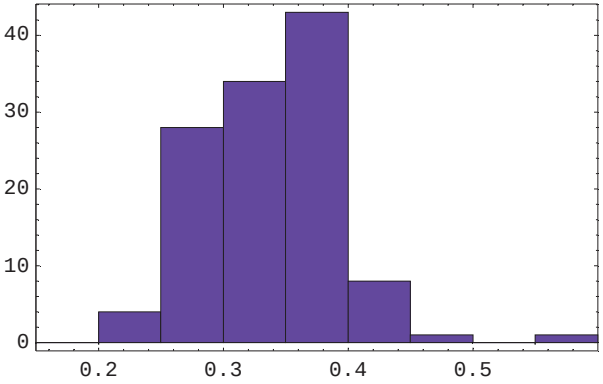

```
xplot11 = Histogram[fractions1[[1]], Frame → True,  HistogramRange → {0.16, 0.58}, BarStyle → {RGBColor[1, 0, 0]}];
```



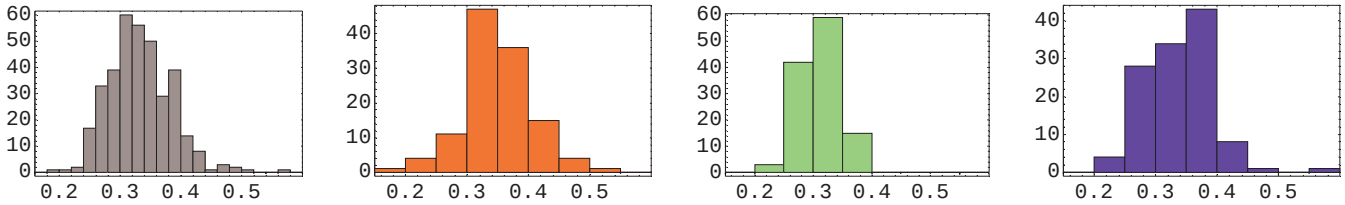
```
xplot12 = Histogram[fractions1[[2]], Frame → True,  HistogramRange → {0.16, 0.58}, BarStyle → {RGBColor[0, 1, 0]}];
```




```
xplot13 = Histogram[fractions1[[3]], Frame → True,  HistogramRange → {0.16, 0.58}, BarStyle → {RGBColor[0, 0, 1]}];
```




```
Show[GraphicsArray[{xplot1, xplot11, xplot12, xplot13}]];
```




```
Map[Mean, fractions1]
```

```
{0.350757, 0.310105, 0.339139}
```

```
Mean[Flatten[fractions1]]
```

```
0.333333
```

```
DispersionReport[Flatten[fractions1]]
```

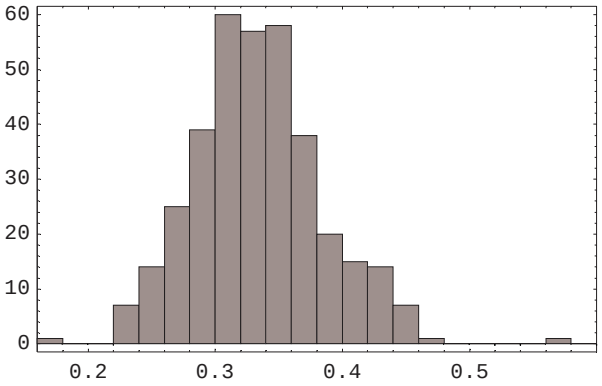
```
{Variance → 0.00264727, StandardDeviation → 0.0514516, SampleRange → 0.365744,
 MeanDeviation → 0.0405669, MedianDeviation → 0.033445, QuartileDeviation → 0.0333316}
```

```
stats1 = Map[DispersionReport, fractions1] // N;
TableForm[Transpose[stats1]]
```

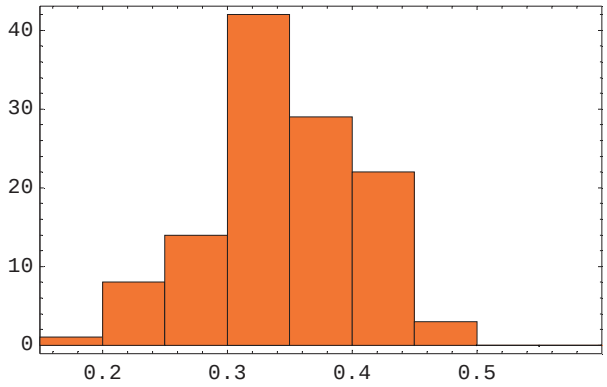
Variance → 0.0032052	Variance → 0.00107722	Variance → 0.00281999
StandardDeviation → 0.0566145	StandardDeviation → 0.032821	StandardDeviation → 0.0531036
SampleRange → 0.317917	SampleRange → 0.141884	SampleRange → 0.332552
MeanDeviation → 0.0444637	MeanDeviation → 0.0272724	MeanDeviation → 0.0437443
MedianDeviation → 0.0373626	MedianDeviation → 0.0253346	MedianDeviation → 0.0416558
QuartileDeviation → 0.0373021	QuartileDeviation → 0.0256767	QuartileDeviation → 0.0397454

■ uncorrected average tapping data

```
data1b = Drop[pid9048x06avg, 1]; (* remove pickup note *)
beats1bcor = Drop[RotateLeft[pid9048x06avg, 1] - pid9048x06avg, -1];
beats1bcor = Drop[beats1bcor, -3];(* remove last measure *)

measure1bdurs = Map[Apply[Plus, #] &, Partition[beats1bcor, 3]];

newdata1b = Transpose[{measure1bdurs, Partition[beats1bcor, 3]}];
fractions1b = Transpose[Map[{N[#[[2]][[1]] / #[[1]]], N[#[[2]][[2]] / #[[1]] ],
        N[#[[2]][[3]] / #[[1]] ]} &, newdata1b]];

xplot1b = Histogram[Flatten[fractions1b], Frame → True, HistogramRange → {0.16, 0.58}, BarStyle → {RGBColor[0.5, 0.5, 0.5]}];
```



```
xplot1b1 = Histogram[fractions1b[[1]], Frame → True,  HistogramRange → {0.16, 0.58}, BarStyle → {RGBColor[1, 0, 0]}];
```

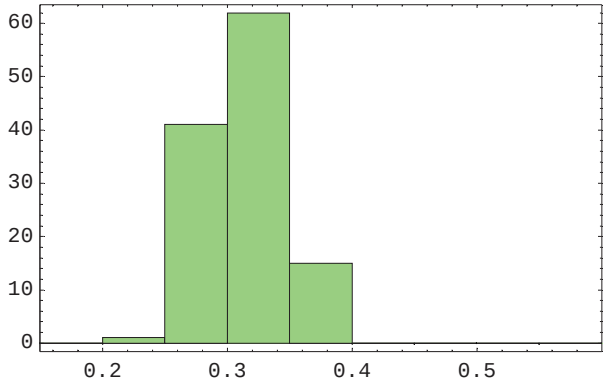


000oooo0?ooo`0]0?ooo`040000003o

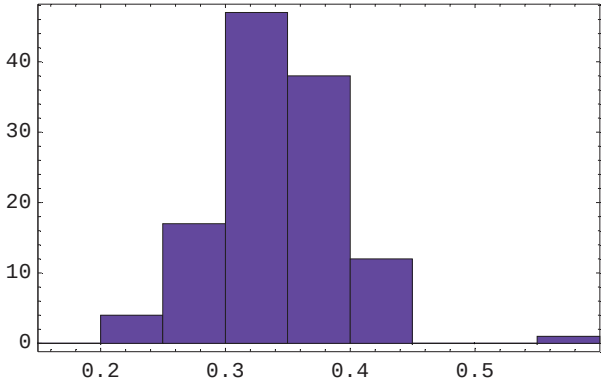
ool0oooo00000080oooo00@000000?ooo`3oool000000P3oool010000000oooo0?ooo`0000090?oo

o`00C@3oool2000000@0oooo0P0000040?ooo`800000=03oool2000003P0oooo0P0000040?ooo`80

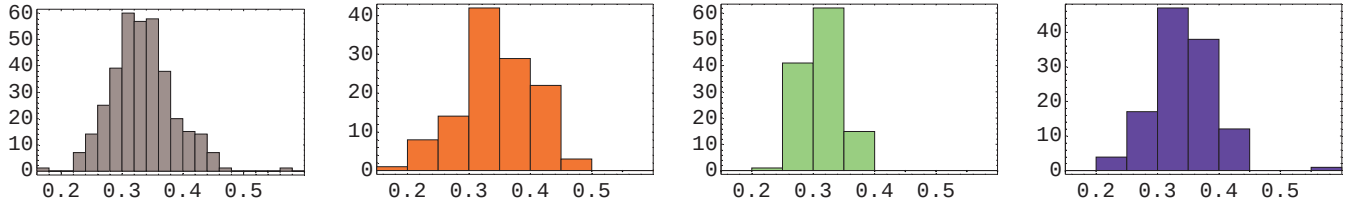
0000103oool200000340ooo



```
xplot1b3 = Histogram[fractions1b[[3]], Frame → True,  HistogramRange → {0.16, 0.58}, BarStyle → {RGBColor[0, 0, 1]}];
```




```
Show[GraphicsArray[{xplot1b, xplot1b1, xplot1b2, xplot1b3}]];
```




```
Map[Mean, fractions1b]
```

```
{0.346804, 0.312709, 0.340488}
```

```
Mean[Flatten[fractions1b]]
```

```
0.333333
```

```
DispersionReport[Flatten[fractions1b]]
```

```
{Variance → 0.00257596, StandardDeviation → 0.0507539, SampleRange → 0.400551,
 MeanDeviation → 0.0394516, MedianDeviation → 0.0312946, QuartileDeviation → 0.0306724}
```

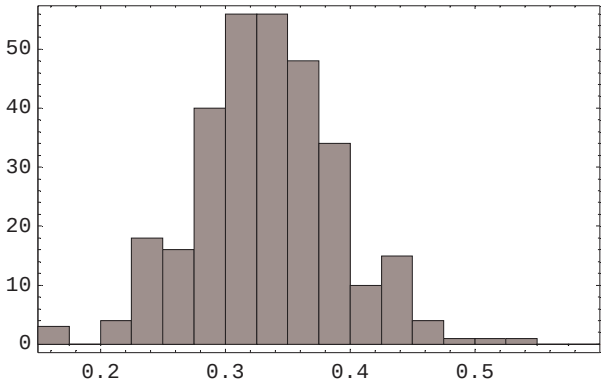
```
 stats1b = Map[DispersionReport, fractions1b] // N;
TableForm[Transpose[stats1b]]
```

Variance → 0.00349965	Variance → 0.00101334	Variance → 0.00259495
StandardDeviation → 0.0591578	StandardDeviation → 0.031833	StandardDeviation → 0.0509406
SampleRange → 0.307544	SampleRange → 0.159168	SampleRange → 0.335608
MeanDeviation → 0.0479919	MeanDeviation → 0.0268304	MeanDeviation → 0.0388674
MedianDeviation → 0.0409272	MedianDeviation → 0.025614	MedianDeviation → 0.0318784
QuartileDeviation → 0.0418094	QuartileDeviation → 0.0266339	QuartileDeviation → 0.0303005

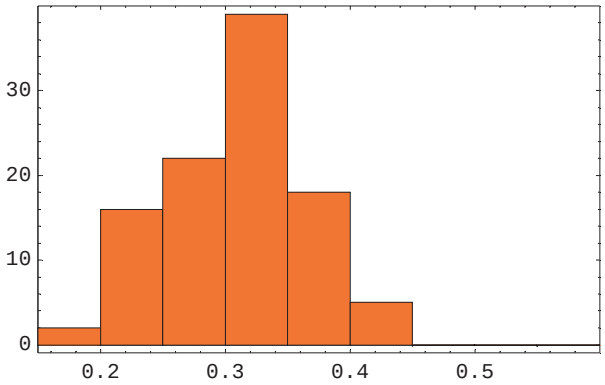
■ Mazurka in A minor, Op. 7, No. 2; Friedman 1930

■ corrected timing data

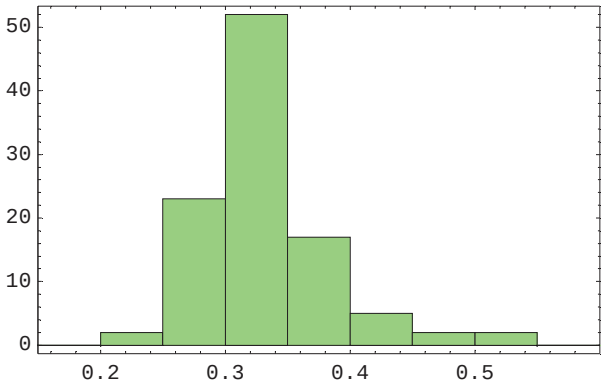
```
data2 = Drop[pid5667230x09corrected, 1]; (* remove pickup note *)
beats2cor = Drop[RotateLeft[pid5667230x09corrected, 1] - pid5667230x09corrected, -1];
beats2cor = Drop[beats2cor, -3];(* remove last measure *)

measure2durs = Map[Apply[Plus, #] &, Partition[beats2cor, 3]];

newdata2 = Transpose[{measure2durs, Partition[beats2cor, 3]}];
fractions2 = Transpose[Map[{N[#[[2]][[1]] / #[[1]]], N[#[[2]][[2]] / #[[1]] ],
       N[#[[2]][[3]] / #[[1]] ]} &, newdata2]];

xplot2 = Histogram[Flatten[fractions2], Frame → True, HistogramRange → {0.16, 0.58}, BarStyle → {RGBColor[0.5, 0.5, 0.5]}];
```



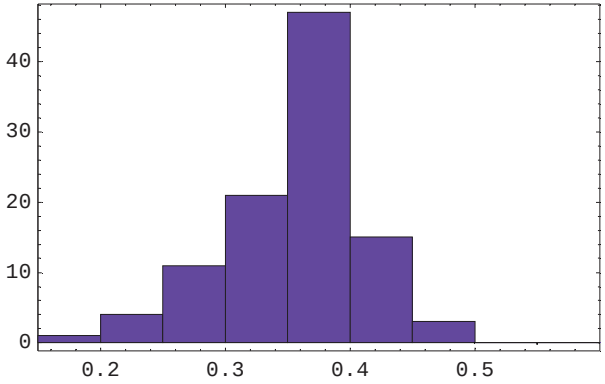

```
xplot21 = Histogram[fractions2[[1]], Frame → True,  HistogramRange → {0.16, 0.58}, BarStyle → {RGBColor[1, 0, 0]}];
```



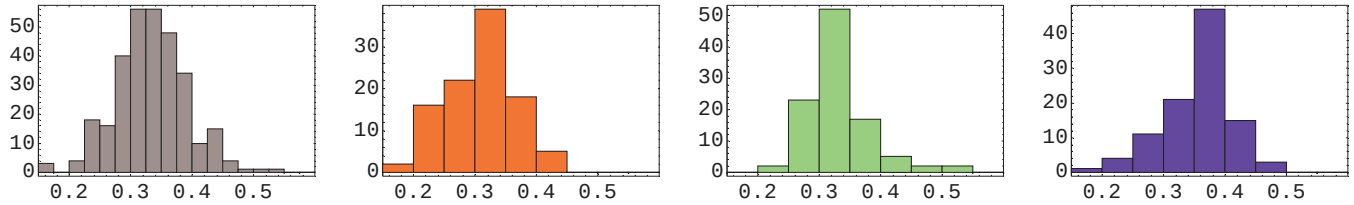
```
xplot22 = Histogram[fractions2[[2]], Frame → True,  HistogramRange → {0.16, 0.58}, BarStyle → {RGBColor[0, 1, 0]}];
```




```
xplot23 = Histogram[fractions2[[3]], Frame → True,  HistogramRange → {0.16, 0.58}, BarStyle → {RGBColor[0, 0, 1]}];
```




```
Show[GraphicsArray[{xplot2, xplot21, xplot22, xplot23}]];
```




```
Map[Mean, fractions2]
```

```
{0.308194, 0.331465, 0.360341}
```

```
DispersionReport[Flatten[fractions2]]
```

```
{Variance → 0.00376596, StandardDeviation → 0.0613674, SampleRange → 0.502754,
 MeanDeviation → 0.0462326, MedianDeviation → 0.034741, QuartileDeviation → 0.0367049}
```

```
Mean[Flatten[fractions2]]
```

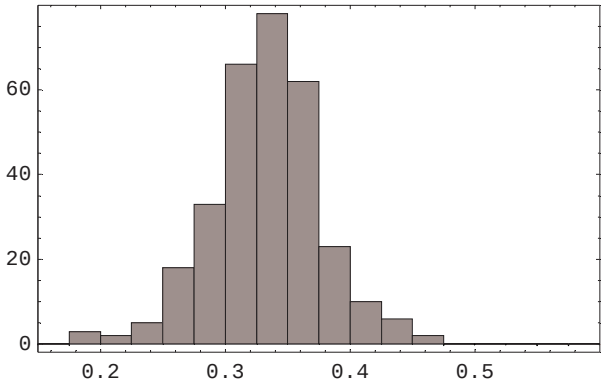
```
0.333333
```

```
stats2 = Map[DispersionReport, fractions2] // N;
TableForm[Transpose[stats2]]
```

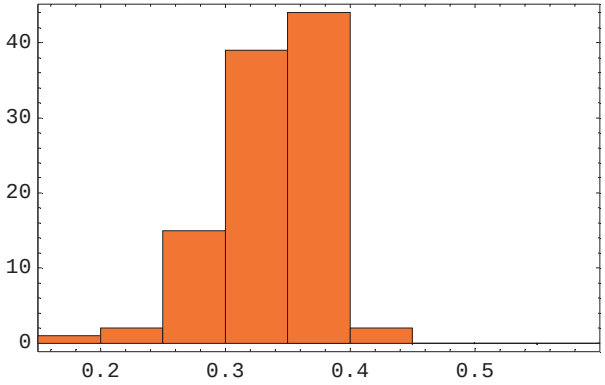
Variance → 0.00325751	Variance → 0.00277782	Variance → 0.00395813
StandardDeviation → 0.0570746	StandardDeviation → 0.0527051	StandardDeviation → 0.0629137
SampleRange → 0.297172	SampleRange → 0.328034	SampleRange → 0.475915
MeanDeviation → 0.0430265	MeanDeviation → 0.0382406	MeanDeviation → 0.0445349
MedianDeviation → 0.0346942	MedianDeviation → 0.024636	MedianDeviation → 0.0301536
QuartileDeviation → 0.0315148	QuartileDeviation → 0.0252668	QuartileDeviation → 0.0303891

■ uncorrected average tapping data

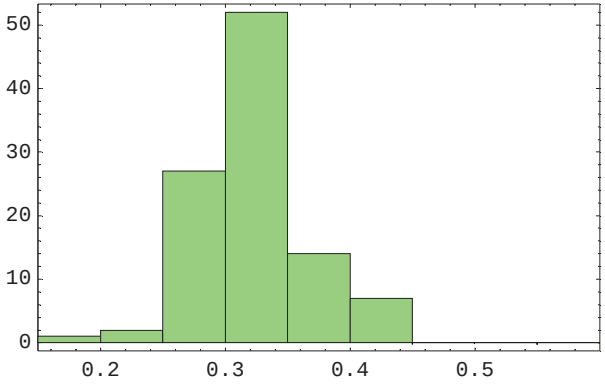
```
data2b = Drop[pid5667230x09avg, 1]; (* remove pickup note *)
beats2bcor = Drop[RotateLeft[pid5667230x09avg, 1] - pid5667230x09avg, -1];
beats2bcor = Drop[beats2bcor, -3];(* remove last measure *)

measure2bdurs = Map[Apply[Plus, #] &, Partition[beats2bcor, 3]];

newdata2b = Transpose[{measure2bdurs, Partition[beats2bcor, 3]}];
fractions2b = Transpose[Map[{N[#[[2]][[1]] / #[[1]]], N[#[[2]][[2]] / #[[1]] ],
       N[#[[2]][[3]] / #[[1]] ]} &, newdata2b]];

xplot2b = Histogram[Flatten[fractions2b], Frame → True, HistogramRange → {0.16, 0.58}, BarStyle → {RGBColor[0.5, 0.5, 0.5]}];
```



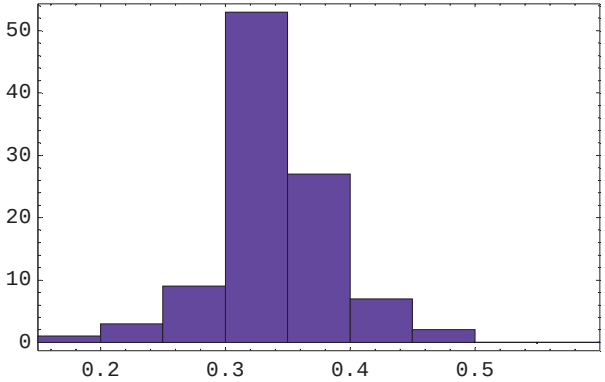
```
xplot2b1 = Histogram[fractions2b[[1]], Frame → True,  HistogramRange → {0.16, 0.58}, BarStyle → {RGBColor[1, 0, 0]}];
```



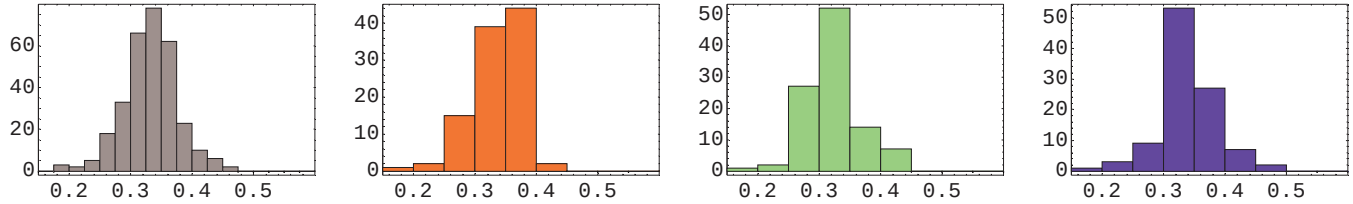

```
xplot2b2 = Histogram[fractions2b[[2]], Frame → True,  HistogramRange → {0.16, 0.58}, BarStyle → {RGBColor[0, 1, 0]}];
```




```
xplot2b3 = Histogram[fractions2b[[3]], Frame → True,  HistogramRange → {0.16, 0.58}, BarStyle → {RGBColor[0, 0, 1]}];
```



```
Show[GraphicsArray[{xplot2b, xplot2b1, xplot2b2, xplot2b3}]];
```



```
Map[Mean, fractions2b]
```

```
{0.33852, 0.320177, 0.341303}
```

```
Mean[Flatten[fractions2b]]
```

```
0.333333
```

```
DispersionReport[Flatten[fractions2b]]
```

```
{Variance → 0.00228893, StandardDeviation → 0.0478428, SampleRange → 0.464096,
 MeanDeviation → 0.0346952, MedianDeviation → 0.0259774, QuartileDeviation → 0.0269128}
```

```
stats2b = Map[DispersionReport, fractions2b] // N;
TableForm[Transpose[stats2b]]
```

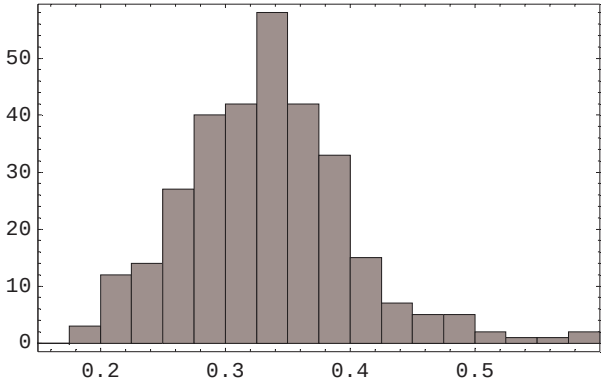
Variance → 0.00170569	Variance → 0.00190778	Variance → 0.00303213
StandardDeviation → 0.0413001	StandardDeviation → 0.0436781	StandardDeviation → 0.0550648
SampleRange → 0.257722	SampleRange → 0.26428	SampleRange → 0.447175
MeanDeviation → 0.0312391	MeanDeviation → 0.0322745	MeanDeviation → 0.0354967
MedianDeviation → 0.0197427	MedianDeviation → 0.0196477	MedianDeviation → 0.0210443
QuartileDeviation → 0.0253983	QuartileDeviation → 0.0229994	QuartileDeviation → 0.021588

■ Mazurka in F minor, Op. 7, No. 3; Friedman 1930

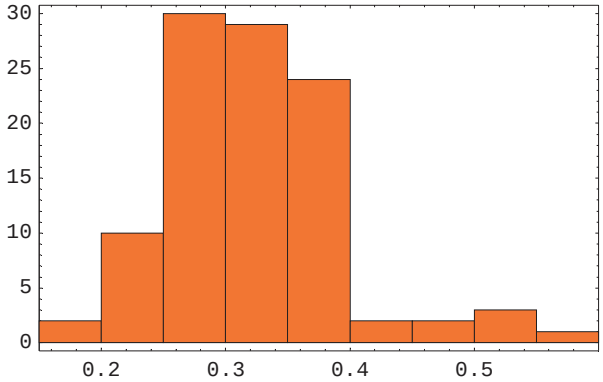
■ corrected timing data

```
beats3cor = Drop[RotateLeft[pid5667230x10corrected, 1] - pid5667230x10corrected, -1];
beats3cor = Drop[beats3cor, -3];(* remove last measure *)

measure3durs = Map[Apply[Plus, #] &, Partition[beats3cor, 3]];

newdata3 = Transpose[{measure3durs, Partition[beats3cor, 3]}];
fractions3 = Transpose[Map[{N[#[[2]][[1]] / #[[1]]], N[#[[2]][[2]] / #[[1]] ],
       N[#[[2]][[3]] / #[[1]] ]} &, newdata3]];
```

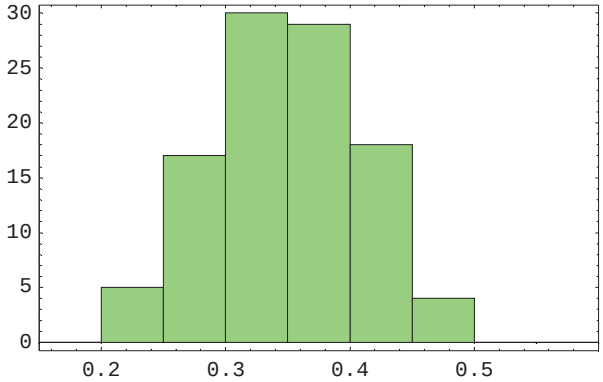
```
xplot3 = Histogram[Flatten[fractions3], Frame → True, HistogramRange → {0.16, 0.58}, BarStyle → {RGBColor[0.5, 0.5, 0.5]}];
```



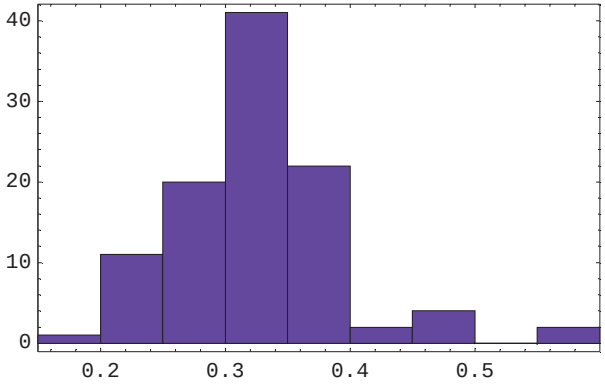

```
xplot31 = Histogram[fractions3[[1]], Frame → True,  HistogramRange → {0.16, 0.58}, BarStyle → {RGBColor[1, 0, 0]}];
```



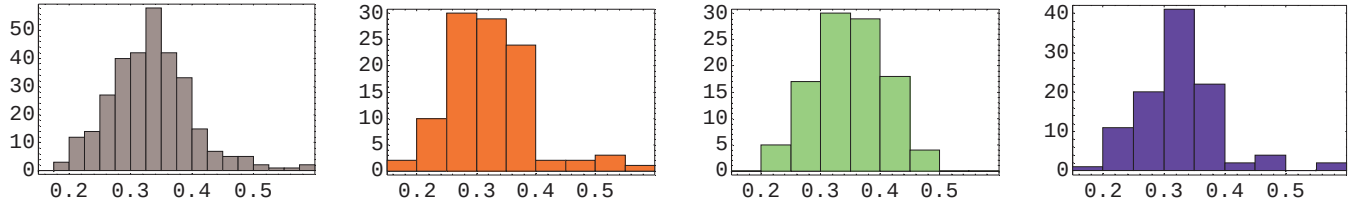

```
xplot32 = Histogram[fractions3[[2]], Frame → True,  HistogramRange → {0.16, 0.58}, BarStyle → {RGBColor[0, 1, 0]}];
```



```
xplot33 = Histogram[fractions3[[3]], Frame → True,  HistogramRange → {0.16, 0.58}, BarStyle → {RGBColor[0, 0, 1]}];
```




```
Show[GraphicsArray[{xplot3, xplot31, xplot32, xplot33}]];
```




```
Map[Mean, fractions3]
```

```
{0.323139, 0.347695, 0.329166}
```

```
Mean[Flatten[fractions3]]
```

```
0.333333
```

```
DispersionReport[Flatten[fractions3]]
```

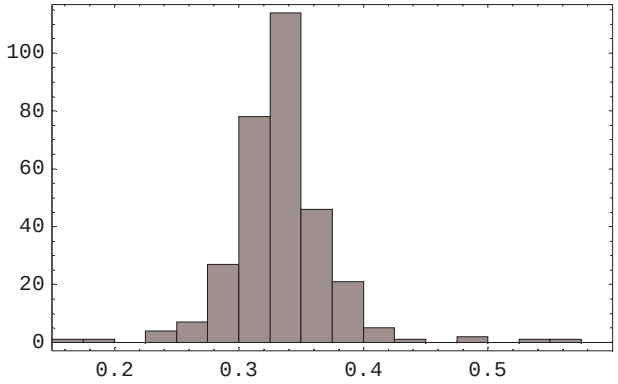
```
{Variance → 0.00447041, StandardDeviation → 0.0668612, SampleRange → 0.405042,
 MeanDeviation → 0.0502984, MedianDeviation → 0.04041, QuartileDeviation → 0.0400493}
```

```
stats3 = Map[DispersionReport, fractions3] // N;
TableForm[Transpose[stats3]]
```

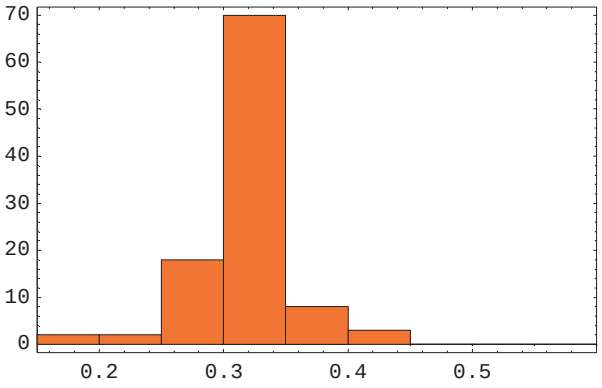
Variance → 0.00494417	Variance → 0.00350757	Variance → 0.00471639
StandardDeviation → 0.0703148	StandardDeviation → 0.0592247	StandardDeviation → 0.068676
SampleRange → 0.374475	SampleRange → 0.277318	SampleRange → 0.405042
MeanDeviation → 0.0524764	MeanDeviation → 0.047732	MeanDeviation → 0.0480444
MedianDeviation → 0.0466148	MedianDeviation → 0.0431934	MedianDeviation → 0.0361549
QuartileDeviation → 0.0418633	QuartileDeviation → 0.042615	QuartileDeviation → 0.0359858

■ uncorrected average tapping data

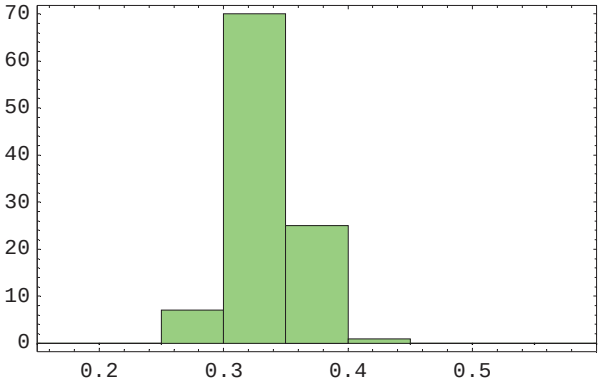
```
data3b = Drop[pid5667230x10avg, 1]; (* remove pickup note *)
beats3bcor = Drop[RotateLeft[pid5667230x10avg, 1] - pid5667230x10avg, -1];
beats3bcor = Drop[beats3bcor, -3];(* remove last measure *)

measure3bdurs = Map[Apply[Plus, #] &, Partition[beats3bcor, 3]];

newdata3b = Transpose[{measure3bdurs, Partition[beats3bcor, 3]}];
fractions3b = Transpose[Map[{N[#[[2]][[1]] / #[[1]]], N[#[[2]][[2]] / #[[1]] ],
       N[#[[2]][[3]] / #[[1]] ]} &, newdata3b]];

xplot3b = Histogram[Flatten[fractions3b], Frame → True, HistogramRange → {0.16, 0.58}, BarStyle → {RGBColor[0.5, 0.5, 0.5]}];
```



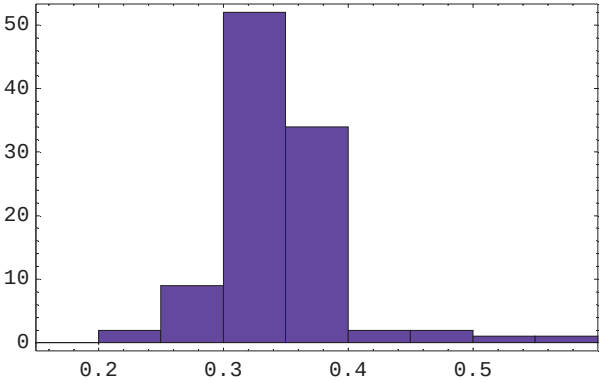

```
xplot3b1 = Histogram[fractions3b[[1]], Frame → True,  HistogramRange → {0.16, 0.58}, BarStyle → {RGBColor[1, 0, 0]}];
```



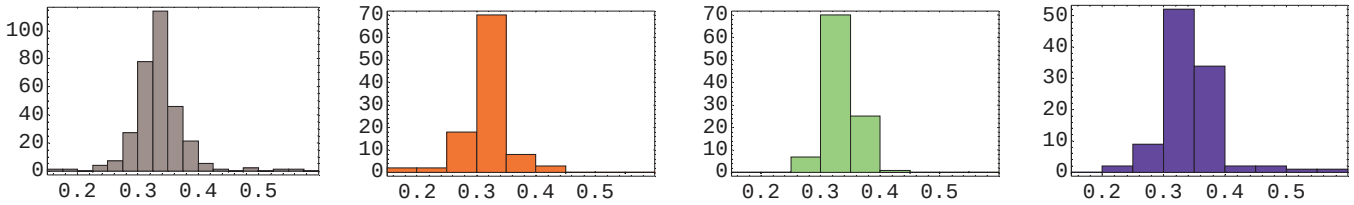
```
xplot3b2 = Histogram[fractions3b[[2]], Frame → True,  HistogramRange → {0.16, 0.58}, BarStyle → {RGBColor[0, 1, 0]}];
```




```
xplot3b3 = Histogram[fractions3b[[3]], Frame → True,  HistogramRange → {0.16, 0.58}, BarStyle → {RGBColor[0, 0, 1]}];
```




```
Show[GraphicsArray[{xplot3b, xplot3b1, xplot3b2, xplot3b3}]];
```




```
Map[Mean, fractions3b]
```

```
{0.318734, 0.334309, 0.346957}
```

```
Mean[Flatten[fractions3b]]
```

```
0.333333
```

```
DispersionReport[Flatten[fractions3b]]
```

```
{Variance → 0.0016036, StandardDeviation → 0.0400449, SampleRange → 0.401831,
 MeanDeviation → 0.026228, MedianDeviation → 0.0176419, QuartileDeviation → 0.0175795}
```

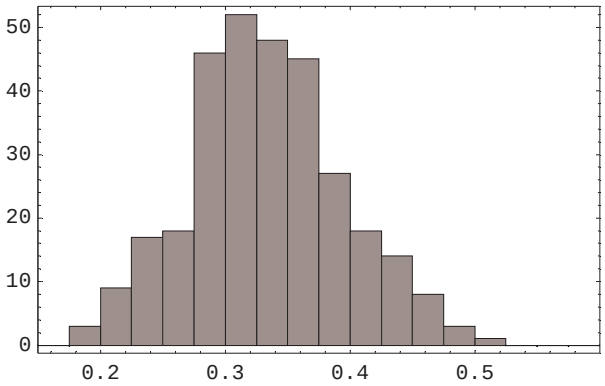
```
stats3b = Map[DispersionReport, fractions3b] // N;
TableForm[Transpose[stats3b]]
```

Variance → 0.00131316	Variance → 0.000660964	Variance → 0.0024645
StandardDeviation → 0.0362375	StandardDeviation → 0.0257092	StandardDeviation → 0.0496437
SampleRange → 0.24465	SampleRange → 0.150127	SampleRange → 0.333141
MeanDeviation → 0.0245583	MeanDeviation → 0.0193881	MeanDeviation → 0.0335713
MedianDeviation → 0.0166147	MedianDeviation → 0.0155217	MedianDeviation → 0.0226506
QuartileDeviation → 0.0176701	QuartileDeviation → 0.015564	QuartileDeviation → 0.0228389

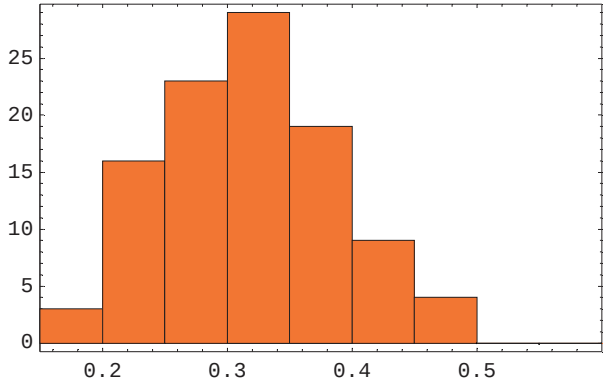
■ Mazurka in F minor, Op. 7, No. 3; Rosen 1989

■ corrected timing data

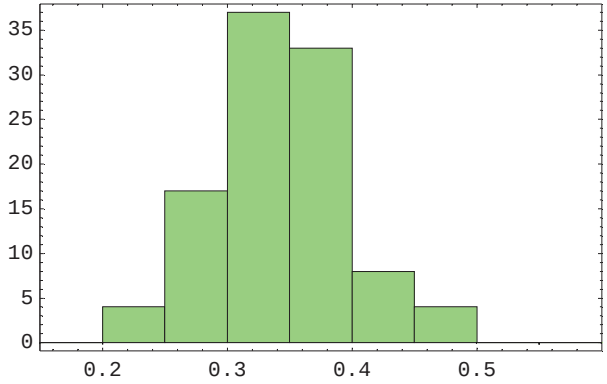
```
beats4cor = Drop[RotateLeft[pid52932x05corrected, 1] - pid52932x05corrected, -1];
beats4cor = Drop[beats4cor, -3];(* remove last measure *)

measure4durs = Map[Apply[Plus, #] &, Partition[beats4cor, 3]];

newdata4 = Transpose[{measure4durs, Partition[beats4cor, 3]}];
fractions4 = Transpose[Map[{N[#[[2]][[1]] / #[[1]]], N[#[[2]][[2]] / #[[1]] ],
       N[#[[2]][[3]] / #[[1]] ]} &, newdata4]];

xplot4 = Histogram[Flatten[fractions4], Frame → True, HistogramRange → {0.16, 0.58}, BarStyle → {RGBColor[0.5, 0.5, 0.5]}];
```



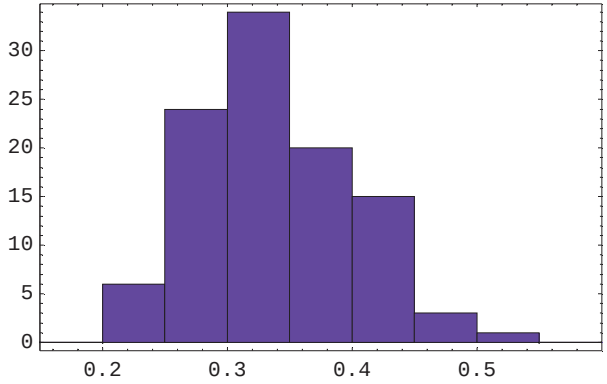
```
xplot41 = Histogram[fractions4[[1]], Frame → True,  HistogramRange → {0.16, 0.58}, BarStyle → {RGBColor[1, 0, 0]}];
```



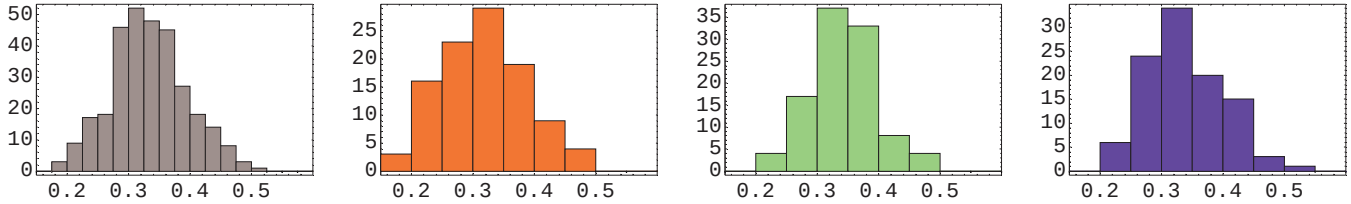

```
xplot42 = Histogram[fractions4[[2]], Frame → True,  HistogramRange → {0.16, 0.58}, BarStyle → {RGBColor[0, 1, 0]}];
```




```
xplot43 = Histogram[fractions4[[3]], Frame → True,  HistogramRange → {0.16, 0.58}, BarStyle → {RGBColor[0, 0, 1]}];
```



```
Show[GraphicsArray[{xplot4, xplot41, xplot42, xplot43}]];
```



```
Map[Mean, fractions4]
```

```
{0.321678, 0.341243, 0.337079}
```

```
Mean[Flatten[fractions4]]
```

```
0.333333
```

```
DispersionReport[Flatten[fractions4]]
```

```
{Variance → 0.00385314, StandardDeviation → 0.0620737, SampleRange → 0.343305,
 MeanDeviation → 0.0491944, MedianDeviation → 0.0393723, QuartileDeviation → 0.0388897}
```

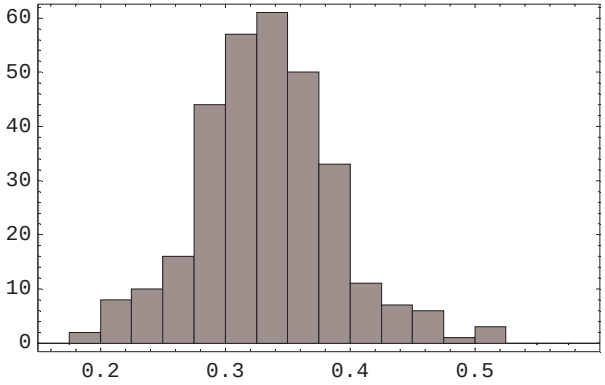
```
stats4 = Map[DispersionReport, fractions4] // N;
TableForm[Transpose[stats4]]
```

```
Variance → 0.00494285          Variance → 0.00259416          Variance → 0.00388346
StandardDeviation → 0.0703054  StandardDeviation → 0.0509329  StandardDeviation → 0.0623174
SampleRange → 0.311217         SampleRange → 0.269642         SampleRange → 0.301875
MeanDeviation → 0.0560369      MeanDeviation → 0.0395009      MeanDeviation → 0.0508214
MedianDeviation → 0.0422233    MedianDeviation → 0.0325842    MedianDeviation → 0.0462949
QuartileDeviation → 0.0435405  QuartileDeviation → 0.0321505  QuartileDeviation → 0.0460295
```

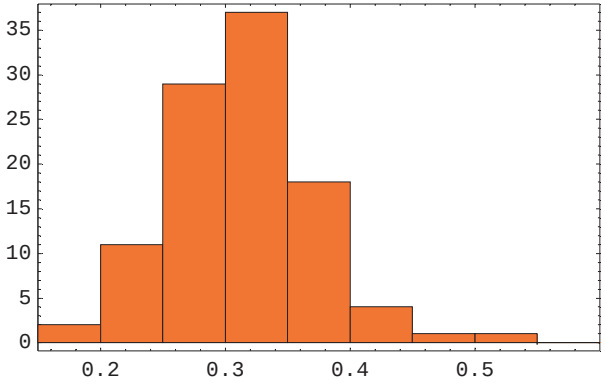
■ uncorrected average tapping data

```
data4b = Drop[pid52932x05avg, 1]; (* remove pickup note *)
beats4bcor = Drop[RotateLeft[pid52932x05avg, 1] - pid52932x05avg, -1];
beats4bcor = Drop[beats4bcor, -3];(* remove last measure *)

measure4bdurs = Map[Apply[Plus, #] &, Partition[beats4bcor, 3]];

newdata4b = Transpose[{measure4bdurs, Partition[beats4bcor, 3]}];
fractions4b = Transpose[Map[{N[#[[2]][[1]] / #[[1]]], N[#[[2]][[2]] / #[[1]] ],
       N[#[[2]][[3]] / #[[1]] ]} &, newdata4b]];
```

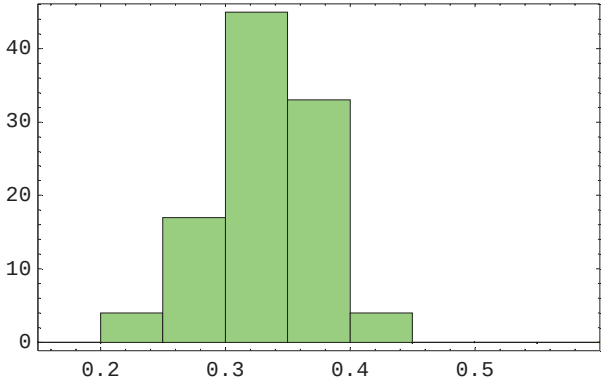
```
xplot4b = Histogram[Flatten[fractions4b], Frame → True, HistogramRange → {0.16, 0.58}, BarStyle → {RGBColor[0.5, 0.5, 0.5]}];
```



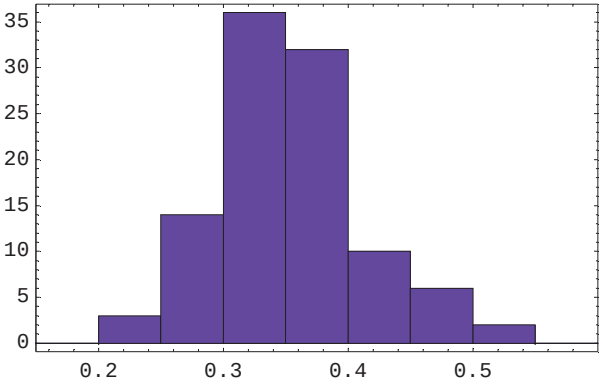

```
xplot4b1 = Histogram[fractions4b[[1]], Frame → True, HistogramRange → {0.16, 0.58}, BarStyle → {RGBColor[1, 0, 0]}];
```



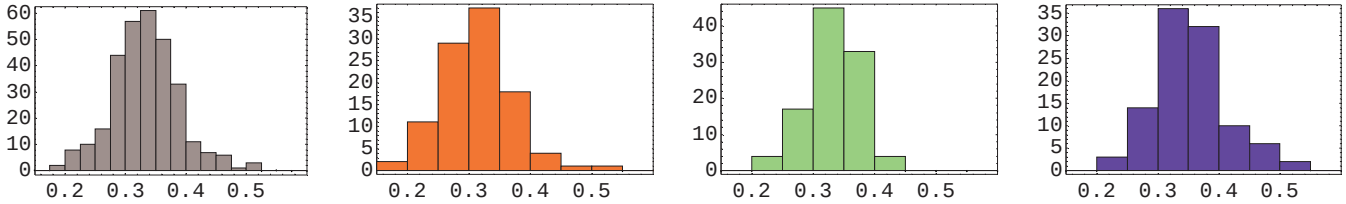

```
xplot4b2 = Histogram[fractions4b[[2]], Frame → True, HistogramRange → {0.16, 0.58}, BarStyle → {RGBColor[0, 1, 0]}];
```



```
xplot4b3 = Histogram[fractions4b[[3]], Frame → True,  HistogramRange → {0.16, 0.58}, BarStyle → {RGBColor[0, 0, 1]}];
```




```
Show[GraphicsArray[{xplot4b, xplot4b1, xplot4b2, xplot4b3}]];
```




```
Map[Mean, fractions4b]
```

```
{0.31286, 0.333546, 0.353594}
```

```
Mean[Flatten[fractions4b]]
```

```
0.333333
```

```
DispersionReport[Flatten[fractions4b]]
```

```
{Variance → 0.00303302, StandardDeviation → 0.0550728, SampleRange → 0.337048,
 MeanDeviation → 0.041517, MedianDeviation → 0.0327341, QuartileDeviation → 0.0331666}
```

```
 stats4b = Map[DispersionReport, fractions4b] // N;
TableForm[Transpose[stats4b]]
```

Variance → 0.00322674	Variance → 0.00172252	Variance → 0.00337145
StandardDeviation → 0.0568044	StandardDeviation → 0.0415032	StandardDeviation → 0.0580642
SampleRange → 0.31795	SampleRange → 0.199991	SampleRange → 0.287131
MeanDeviation → 0.0423968	MeanDeviation → 0.0325055	MeanDeviation → 0.0452183
MedianDeviation → 0.0316901	MedianDeviation → 0.0274517	MedianDeviation → 0.0334332
QuartileDeviation → 0.0318456	QuartileDeviation → 0.026592	QuartileDeviation → 0.0334014

■ All pieces

■ Manually Corrected Data

```
allbeatdurs = Flatten[{fractions1, fractions2, fractions3, fractions4}];

Mean[allbeatdurs]
```

```
0.333333
```

```
DispersionReport[Flatten[{fractions1, fractions2, fractions3, fractions4}]]
```

```
{Variance → 0.00363679, StandardDeviation → 0.0603058, SampleRange → 0.502754,
 MeanDeviation → 0.0463485, MedianDeviation → 0.0370524, QuartileDeviation → 0.0375576}
```

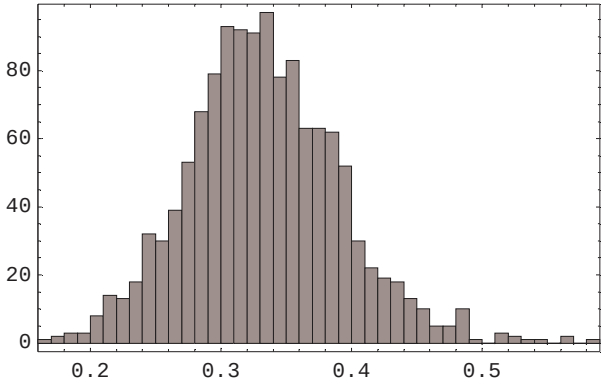
```
allbeat1dur = Flatten[{fractions1[[1]], fractions2[[1]], fractions3[[1]], fractions4[[1]]}];
allbeat2dur = Flatten[{fractions1[[2]], fractions2[[2]], fractions3[[2]], fractions4[[2]]}];
allbeat3dur = Flatten[{fractions1[[3]], fractions2[[3]], fractions3[[3]], fractions4[[3]]}];

Mean[allbeat1dur]
Mean[allbeat2dur]
Mean[allbeat3dur]
```

```
0.32687

0.331785

0.341345
```

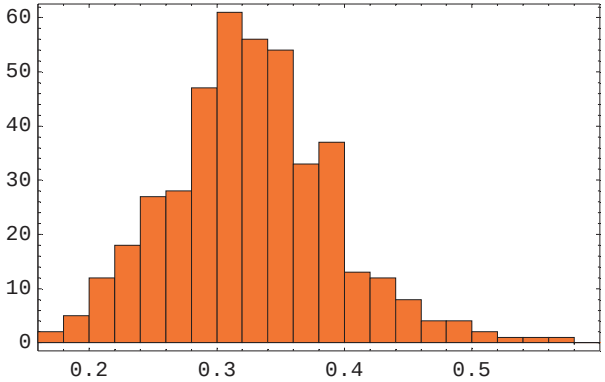
```
DispersionReport[allbeat1dur]
DispersionReport[allbeat2dur]
DispersionReport[allbeat3dur]
```

```
{Variance → 0.00427867, StandardDeviation → 0.0654115, SampleRange → 0.418919,
 MeanDeviation → 0.0502381, MedianDeviation → 0.0394116, QuartileDeviation → 0.0395224}

{Variance → 0.00263245, StandardDeviation → 0.0513074, SampleRange → 0.332226,
 MeanDeviation → 0.0396879, MedianDeviation → 0.0309381, QuartileDeviation → 0.0302262}

{Variance → 0.00390766, StandardDeviation → 0.0625113, SampleRange → 0.475915,
 MeanDeviation → 0.0485149, MedianDeviation → 0.0406842, QuartileDeviation → 0.0410174}
```

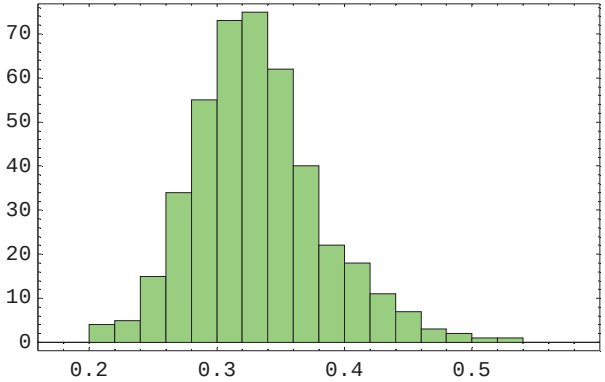
```
xplotall = Histogram[Flatten[allbeatdurs], Frame → True, HistogramRange → {0.16, 0.58}, BarStyle → {RGBColor[0.5, 0.5, 0.5]}];
```



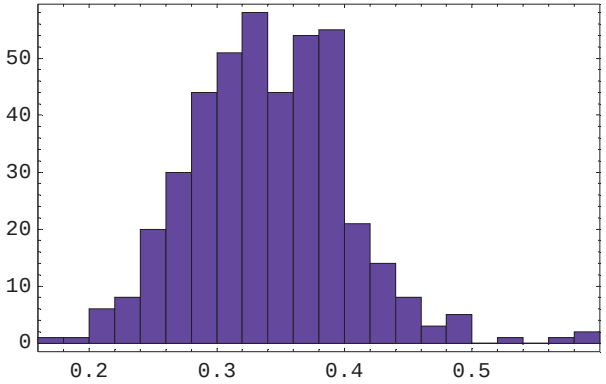

```
xplotallb1 = Histogram[allbeat1dur, Frame → True, HistogramRange → {0.16, 0.58}, BarStyle → {RGBColor[1, 0, 0]}];
```



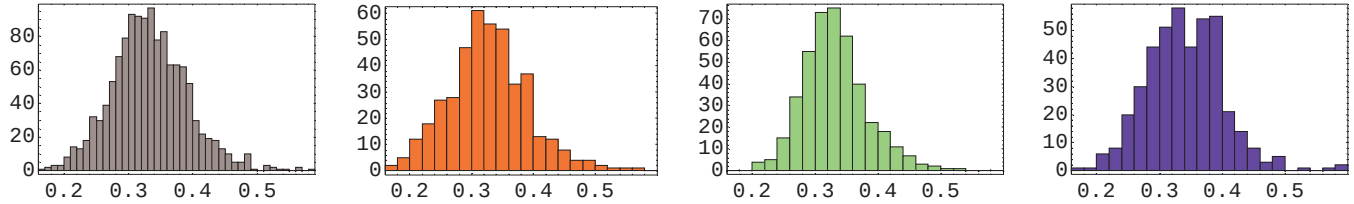

```
xplotallb2 = Histogram[allbeat2dur, Frame → True, HistogramRange → {0.16, 0.58}, BarStyle → {RGBColor[0, 1, 0]}];
```



```
xplotallb3 = Histogram[allbeat3dur, Frame → True,  HistogramRange → {0.16, 0.58}, BarStyle → {RGBColor[0, 0, 1]}];
```




```
Show[GraphicsArray[{xplotall, xplotallb1, xplotallb2, xplotallb3}]];
```



■ Raw Tapping Data

```
allbeatdurstap = Flatten[{fractions1b, fractions2b, fractions3b, fractions4b}];

Mean[allbeatdurstap]
```

```
0.333333
```

```
DispersionReport[Flatten[{fractions1b, fractions2b, fractions3b, fractions4b}]]
```

```
{Variance → 0.00237733, StandardDeviation → 0.0487578, SampleRange → 0.477238,
 MeanDeviation → 0.0356217, MedianDeviation → 0.0270833, QuartileDeviation → 0.0268928}
```

```
allbeat1durt = Flatten[{fractions1b[[1]], fractions2b[[1]], fractions3b[[1]], fractions4b[[1]]}];
allbeat2durt = Flatten[{fractions1b[[2]], fractions2b[[2]], fractions3b[[2]], fractions4b[[2]]}];
allbeat3durt = Flatten[{fractions1b[[3]], fractions2b[[3]], fractions3b[[3]], fractions4b[[3]]}];
```

```
Mean[allbeat1durt]
Mean[allbeat2durt]
Mean[allbeat3durt]
```

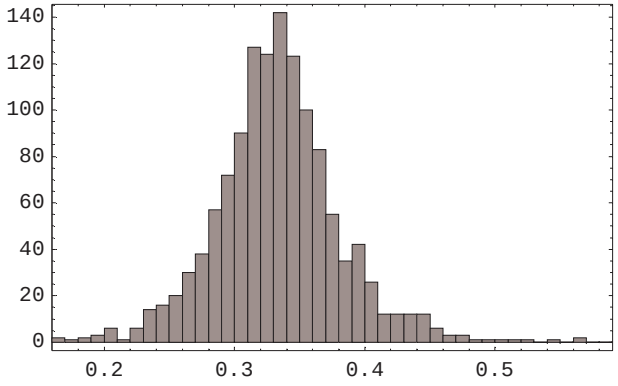
```
0.329886

0.324719

0.345395
```

```
DispersionReport[allbeat1durt]
DispersionReport[allbeat2durt]
DispersionReport[allbeat3durt]
```

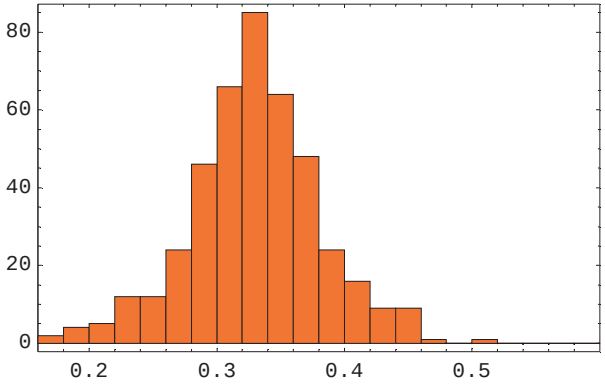
```
{Variance → 0.00265671, StandardDeviation → 0.0515433, SampleRange → 0.339382,
 MeanDeviation → 0.0387175, MedianDeviation → 0.0295679, QuartileDeviation → 0.0296238}

{Variance → 0.00139127, StandardDeviation → 0.0372997, SampleRange → 0.26428,
 MeanDeviation → 0.0288973, MedianDeviation → 0.0235877, QuartileDeviation → 0.0230914}

{Variance → 0.00286303, StandardDeviation → 0.0535073, SampleRange → 0.447175,
 MeanDeviation → 0.0383465, MedianDeviation → 0.0270663, QuartileDeviation → 0.0268572}
```

```
jj
```

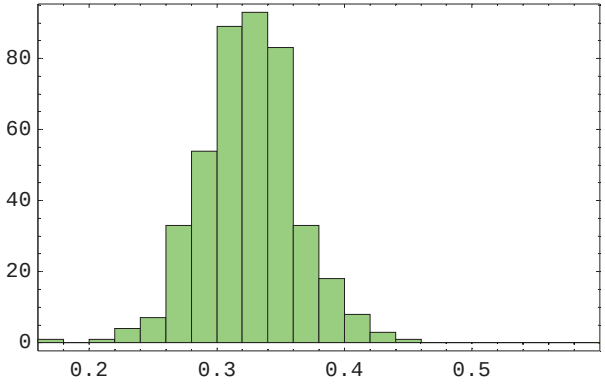
```
xplotalltap = Histogram[Flatten[allbeatdurstap], Frame → True, HistogramRange → {0.16, 0.58}, BarStyle → {RGBColor[0.5, 0.5, 0.5]}];
```



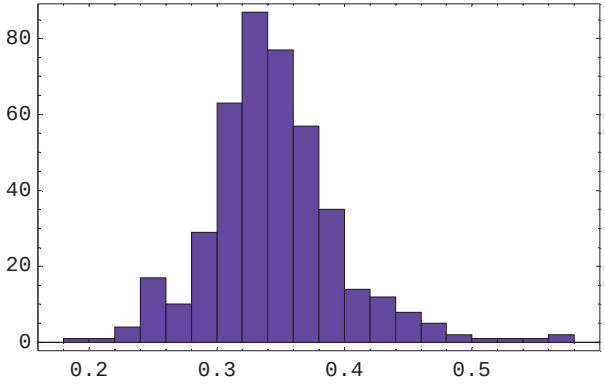
```
xplotallb1t = Histogram[allbeat1durt, Frame → True,  HistogramRange → {0.16, 0.58}, BarStyle → {RGBColor[1, 0, 0]}];
```



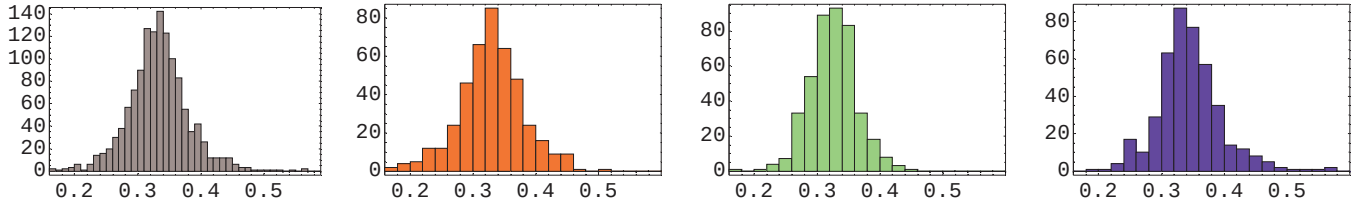

```
xplotallb2t = Histogram[allbeat2durt, Frame → True,  HistogramRange → {0.16, 0.58}, BarStyle → {RGBColor[0, 1, 0]}];
```




```
xplotallb3t = Histogram[allbeat3durt, Frame → True,  HistogramRange → {0.16, 0.58}, BarStyle → {RGBColor[0, 0, 1]}];
```



```
Show[GraphicsArray[{xplotalltap, xplotallb1t, xplotallb2t, xplotallb3t}]];
```



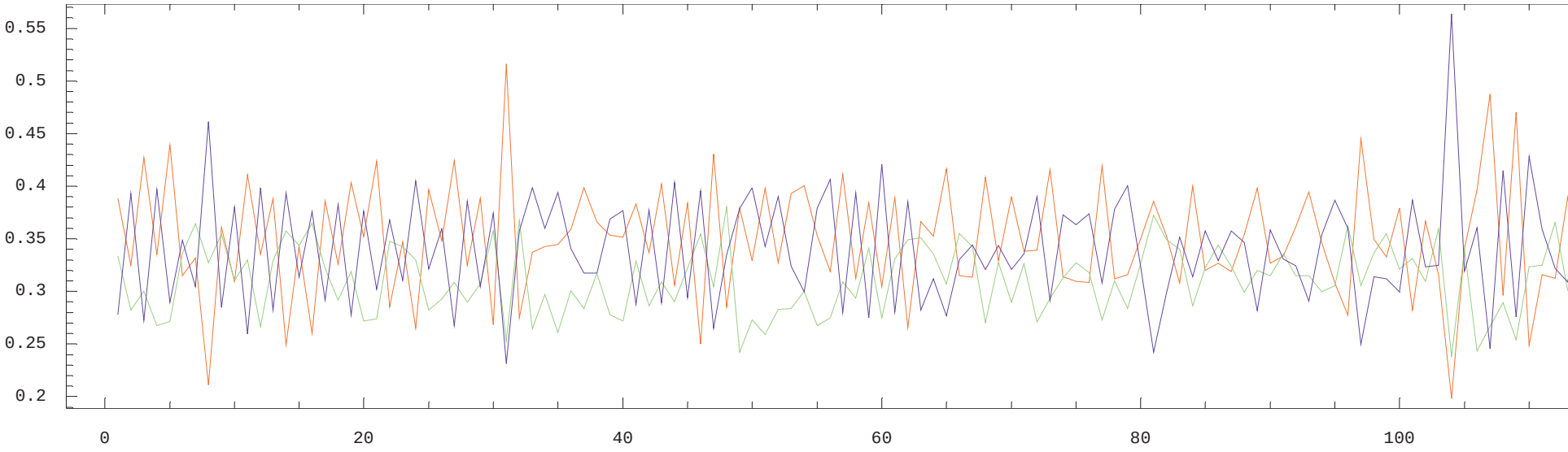
■ Looking at the relative metric fractions of each beat in sequence

```
ShowMetricCycles[list_] := Module[{dplot11, dplot12, dplot13}, Block[{$DisplayFunction = Identity},
   dplot11 = ListPlot[list[[1]], PlotJoined → True, Frame → True, PlotStyle → RGBColor[1, 0, 0], Axes → False];
   dplot12 = ListPlot[list[[2]], PlotJoined → True, Frame → True, PlotStyle → RGBColor[0, 1, 0], Axes → False];
   dplot13 = ListPlot[list[[3]], PlotJoined → True, Frame → True, PlotStyle → RGBColor[0, 0, 1], Axes → False];
  ];
  Show[dplot11, dplot12, dplot13, AspectRatio → 1 / 4]
 ]
```

■ Mazurka in A minor, Op. 7, No. 2 performed by Chiu 1999

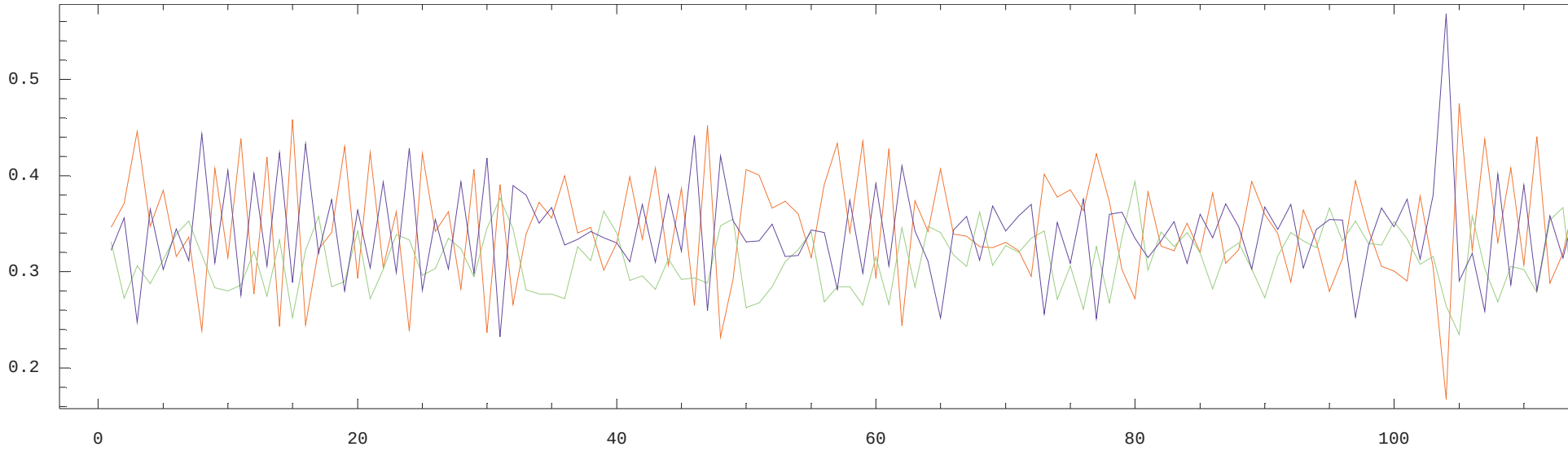
■ Manually Corrected Data

```
ShowMetricCycles[fractions1];
```



■ Raw Tapping Data

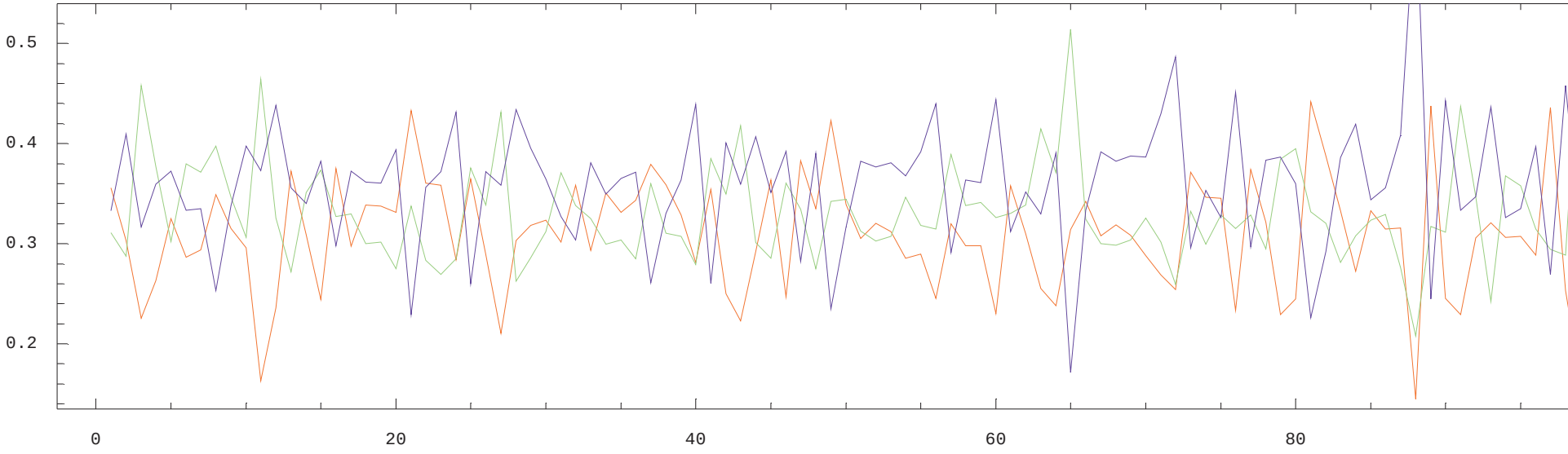
```
ShowMetricCycles[fractions1b];
```



■ Mazurka in A minor, Op. 7, No. 2 performed by Friedman 1930

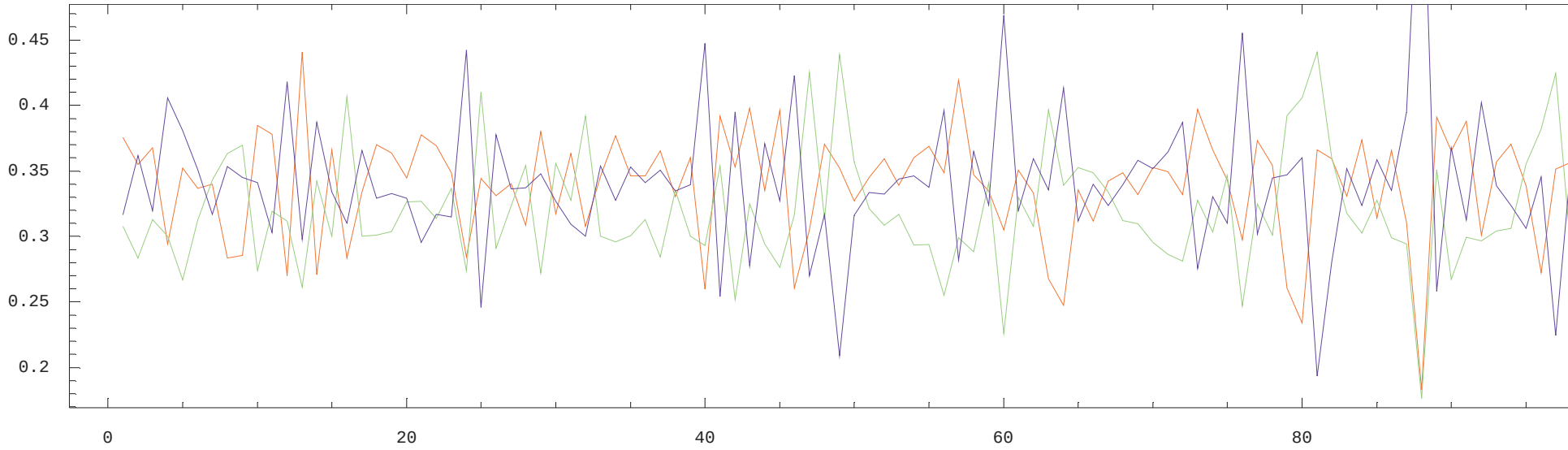
■ Manually Corrected Data

```
ShowMetricCycles[fractions2];
```



■ Raw Tapping Data

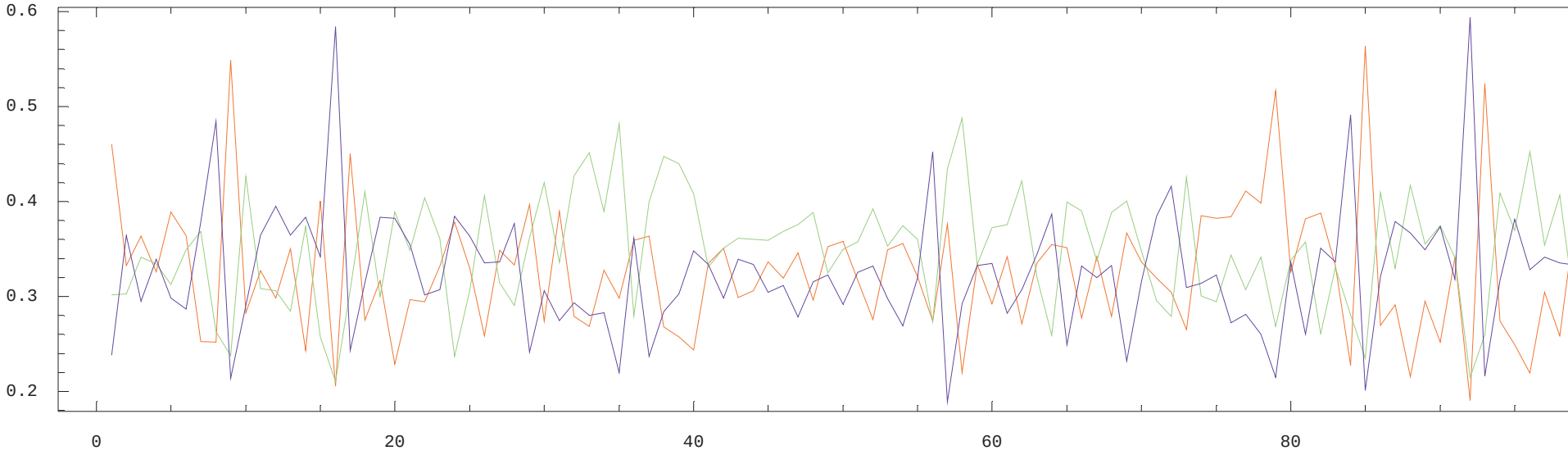
```
ShowMetricCycles[fractions2b];
```



■ Mazurka in F minor, Op. 7, No. 3 performed by Friedman 1930

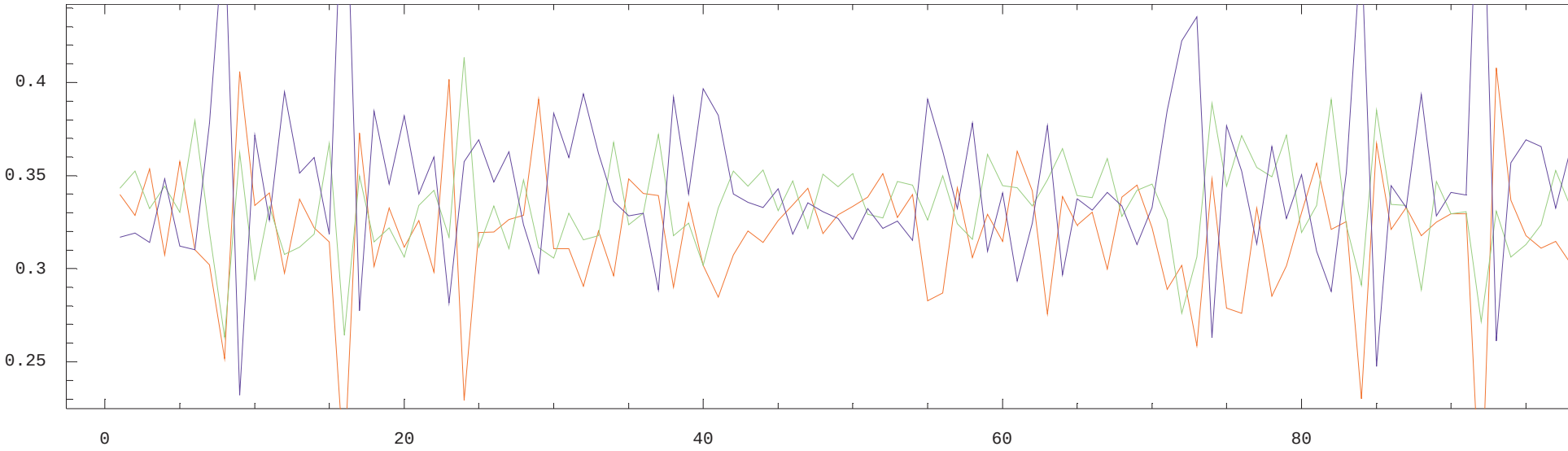
■ Manually Corrected Data

```
ShowMetricCycles[fractions3];
```



■ Raw Tapping Data

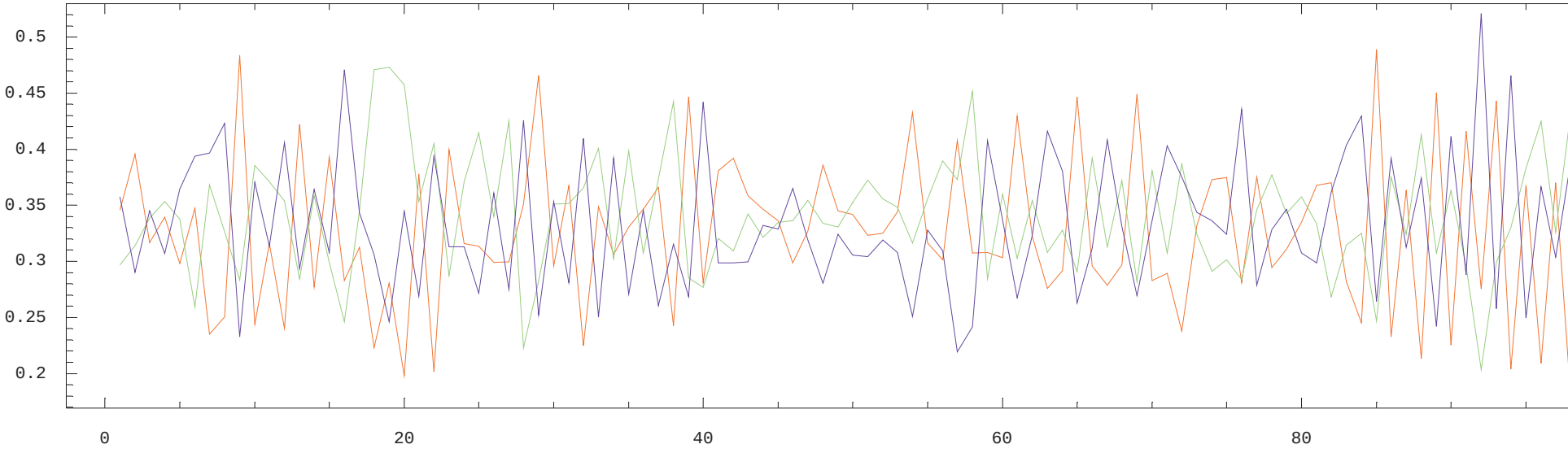
```
ShowMetricCycles[fractions3b];
```



■ Mazurka in F minor, Op. 7, No. 3 performed by Rosen 1989

■ Manually Corrected Data

```
ShowMetricCycles[fractions4];
```



■ Raw Tapping Data

```
ShowMetricCycles[fractions4b];
```