Tempo Change JND

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Raw evaluation data

Data for 20 clicks starting at tempo 60 MM (1000 milliseconds between first two clicks):

```
data2060 = {
   {0.006, 3 / 3},
   {0.005, 10 / 10},
   {0.004, 35 / 40},
   {0.003, 31 / 40},
   {0.0024, 34 / 40},
   {0.002, 25 / 40},
   {0.0015, 28 / 40},
   {0.001, 22 / 40},
   {0.0005, 23 / 40},
   {0.00025, 21 / 40},
   {0.000125, 19 / 40},
   {0.0000625, 4 / 8}
 };
```

Data for 10 clicks starting at tempo 60 MM (1000 milliseconds between first two clicks):

```
data1060 = {
   {0.010, 38 / 40},
   {0.008, 20 / 20},
   {0.006, 37 / 40},
   {0.005, 32 / 40},
   {0.004, 28 / 40},
   {0.003, 29 / 40},
   {0.002, 27 / 40},
   {0.001, 25 / 40},
   {0.0005, 21 / 40}
 };
```

Data for 5 clicks starting at tempo 60 MM (1000 milliseconds between first two clicks):

```
data0560 = {
   {0.016, 39 / 40},
   {0.014, 38 / 40},
   {0.010, 32 / 40},
   {0.008, 30 / 40},
   {0.006, 25 / 40},
   {0.004, 23 / 40}
  };
```

Data for 4 clicks starting at tempo 60 MM (1000 milliseconds between first two clicks):

```
data0460 = {
   {0.025, 19 / 20},
   {0.020, 36 / 40},
   {0.015, 48 / 61},
   {0.012, 32 / 40},
   {0.010, 29 / 40},
   {0.008, 35 / 40},
   {0.006, 27 / 40}
  };
```

Data for 5 clicks starting at tempo 60 MM (1000 milliseconds between first two clicks):

```
data0360 = {
   {0.002, 23 / 40},
   {0.004, 24 / 40},
   {0.006, 24 / 40},
   {0.008, 28 / 40},
   {0.010, 24 / 40},
   {0.015, 26 / 40},
   {0.020, 25 / 40},
   {0.025, 31 / 40},
   {0.030, 30 / 40},
   {0.035, 32 / 40},
   {0.040, 31 / 40},
   {0.045, 38 / 40},
   {0.050, 39 / 40}
  };
```

■ Plot of accuracy v acceleration rate

```
plot2060 = ListPlot[data2060, Frame → True,
  PlotStyle → PointSize[0.02], DisplayFunction → Identity];
plot1060 = ListPlot[data1060, Frame → True,
  PlotStyle → {RGBColor[0, 0, 1], PointSize[0.02]},
  Axes → False, DisplayFunction → Identity];
plot0560 = ListPlot[data0560, Frame → True,
  PlotStyle → {RGBColor[0, 1, 0], PointSize[0.02]},
  Axes → False, DisplayFunction → Identity];
plot0460 = ListPlot[data0460, Frame → True,
  PlotStyle → {RGBColor[1, 0.6, 0], PointSize[0.02]},
  Axes → False, DisplayFunction → Identity];
plot0360 = ListPlot[data0360, Frame → True,
   PlotStyle → {RGBColor[0.75, 0, 0], PointSize[0.02]},
   Axes → False, DisplayFunction → Identity];

Show[plot2060, plot1060, plot0560, plot0360, plot0460, Graphics[
   {Dashing[{0.02, 0.02}], Line[{{0, 0.75}, {0.51, 0.75}}]}],
  PlotRange → {{0, 0.055}, {0.45, 1.}},
  DisplayFunction → $DisplayFunction];
```
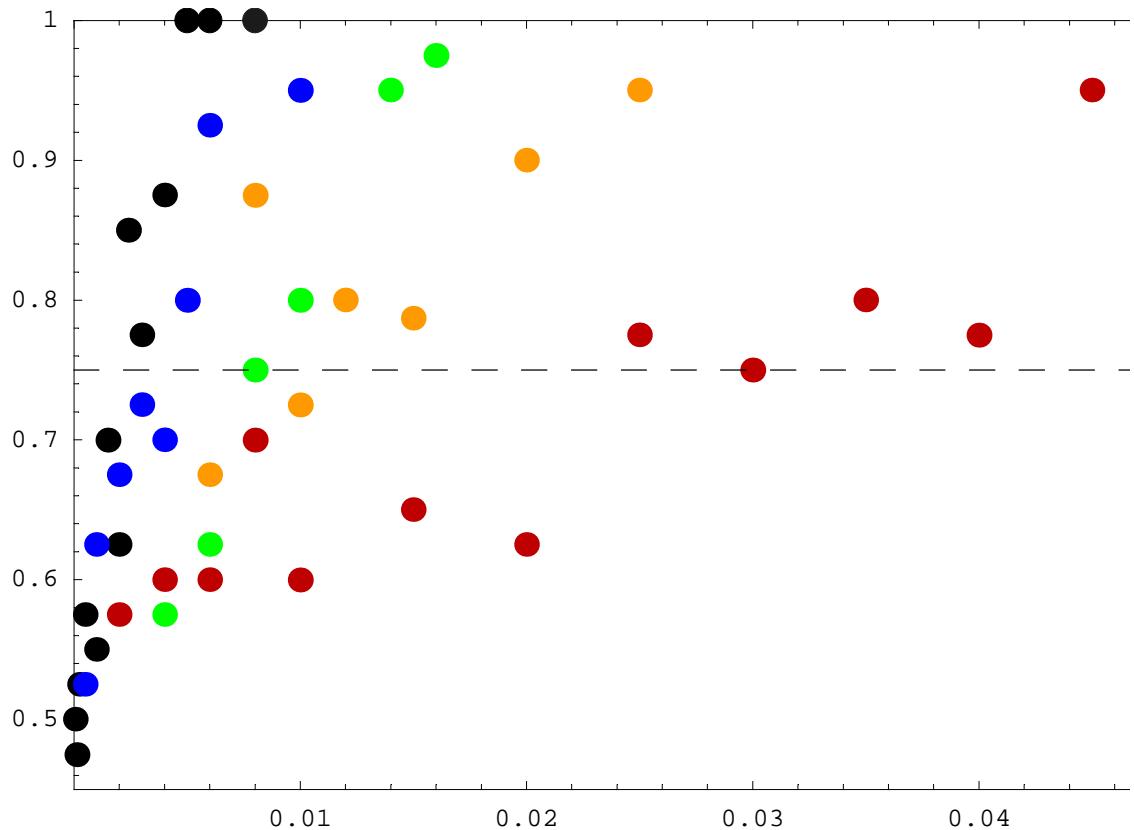
Examining the function of JND rate versus click count

Direct plotting of the data yeilds the following curve:

```
jnds = {{3, 0.0265}, {4, 0.0115},
   {5, 0.0082}, {10, 0.0040}, {20, 0.0024}};

ListPlot[jnds, Frame → True, Axes -> False, PlotJoined → True];
```
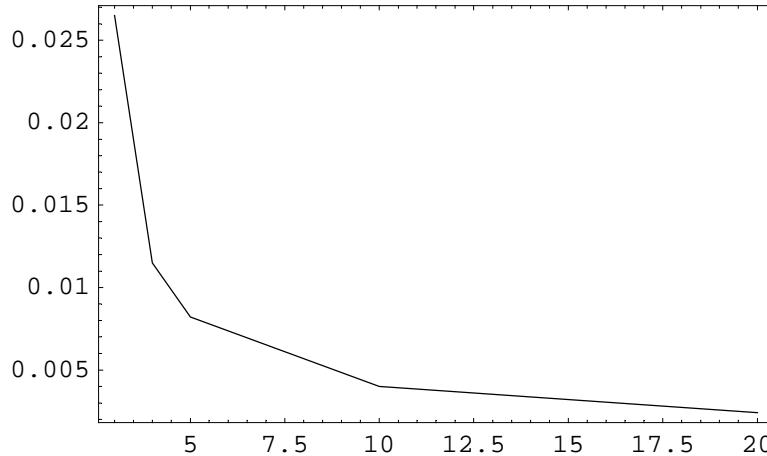

Taking the log of the JND fractions (on the vertical axis):

```
logjnds =
  Transpose[{Transpose[jnds][[1]], Log[2, Transpose[jnds][[2]]]}];
ListPlot[logjnds, Frame → True, Axes → False, PlotJoined → True];
```
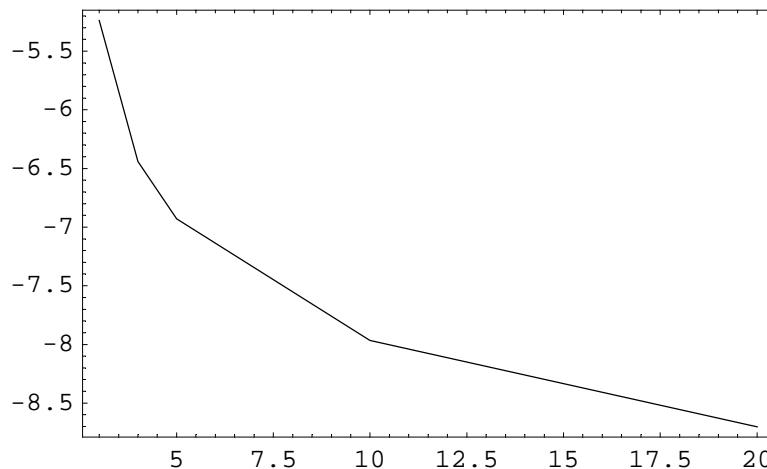

Taking the logarithm of both axes:

```
loglogjnds = Log[10, Map[ {#[[1]], #[[2]]} &, jnds] ] // N;
ListPlot[loglogjnds, Frame → True, Axes → False, PlotJoined → True];
```
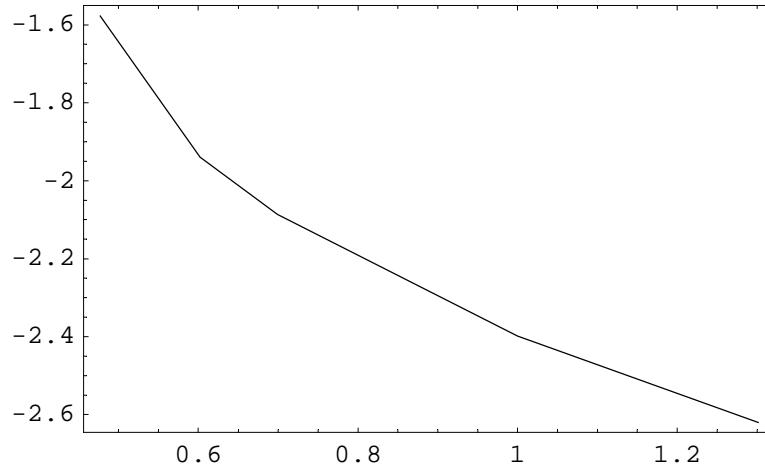

Since the number of clicks is greater than the number of tempo regions, subtract an offset before taking the log of the click counts. An offset
of one converts the clicks into tempo regions (3 clicks = 2 tempo regions):

```
loglogjnds = Log[10, Map[ {#[[1]] - 1, #[[2]]} &, jnds] ] // N;
ListPlot[loglogjnds, Frame → True, Axes → False, PlotJoined → True];
```
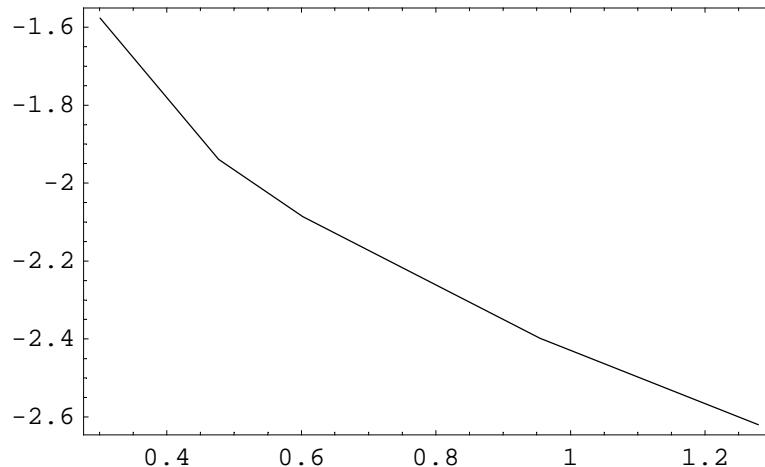

An offset of two removes the first click, and also removes the second click which describes the starting tempo, so an -2 removes the initial tempo from the display, and only counts the changing tempo regions.

```
loglogjnds = Log[10, Map[ {#[[1]] - 2, #[[2]]} &, jnds] ] // N;
ListPlot[loglogjnds, Frame → True, Axes → False, PlotJoined → True];
```
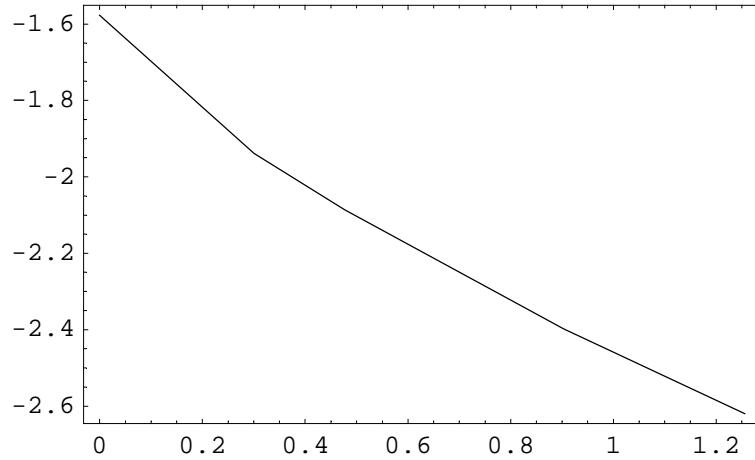
Now examine the offset of -2 and compare to a straight line, since it is fairly close to a line:

```
p0 =
  ListPlot[loglogjnds, Frame → True, Axes → False, PlotJoined → True,
   PlotRange → All, PlotStyle → {Hue[0], Thickness[0.02]}];
```
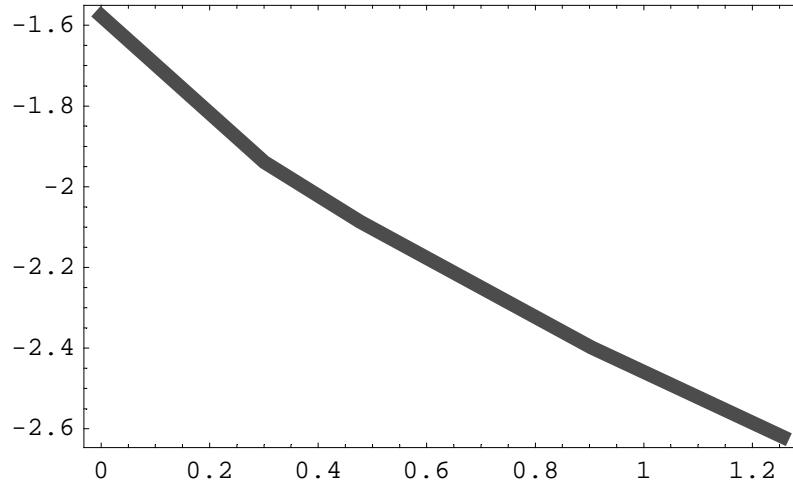
```
p1 = ListPlot[loglogjnds, Frame → True,
   Axes → False, PlotJoined → False, PlotRange → All,
   PlotStyle → {PointSize[0.02], Thickness[0.02]}];
```
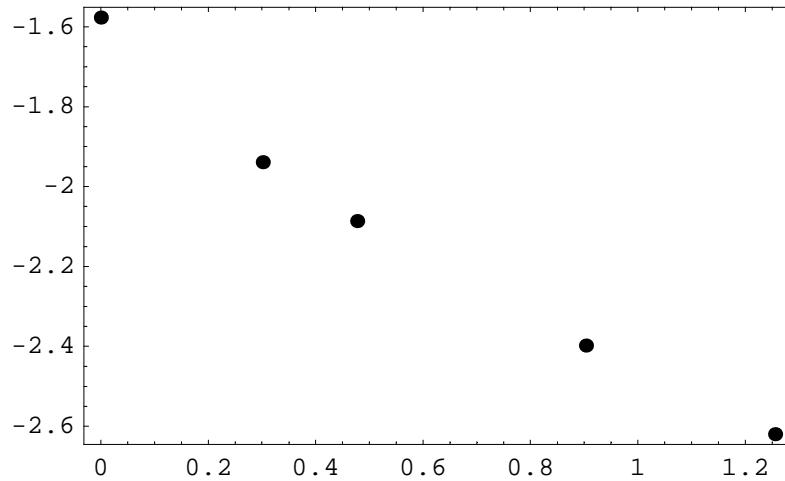


```
fit = Fit[Drop[loglogjnds, 1], {1, x}, x]
```

$-1.73798 - 0.712166\,x$

```
fitp = Plot[fit, {x, 0, 1.25}, Frame → True,
   Axes → False, PlotStyle → Dashing[{0.02, 0.02}]];
```
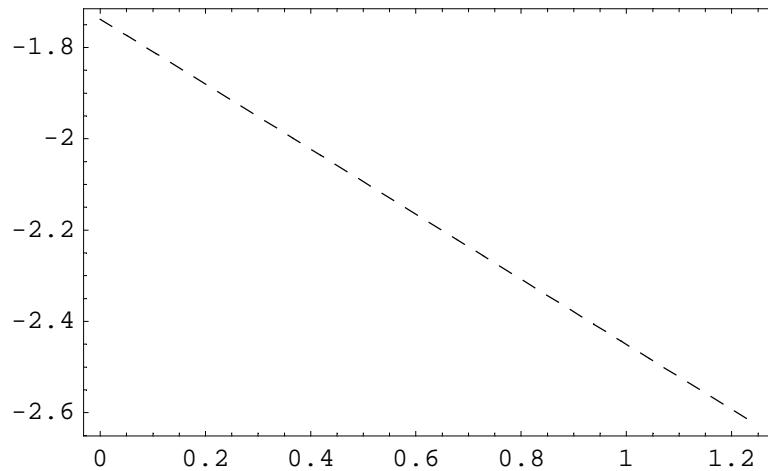
Show[p0, p1, fitp];
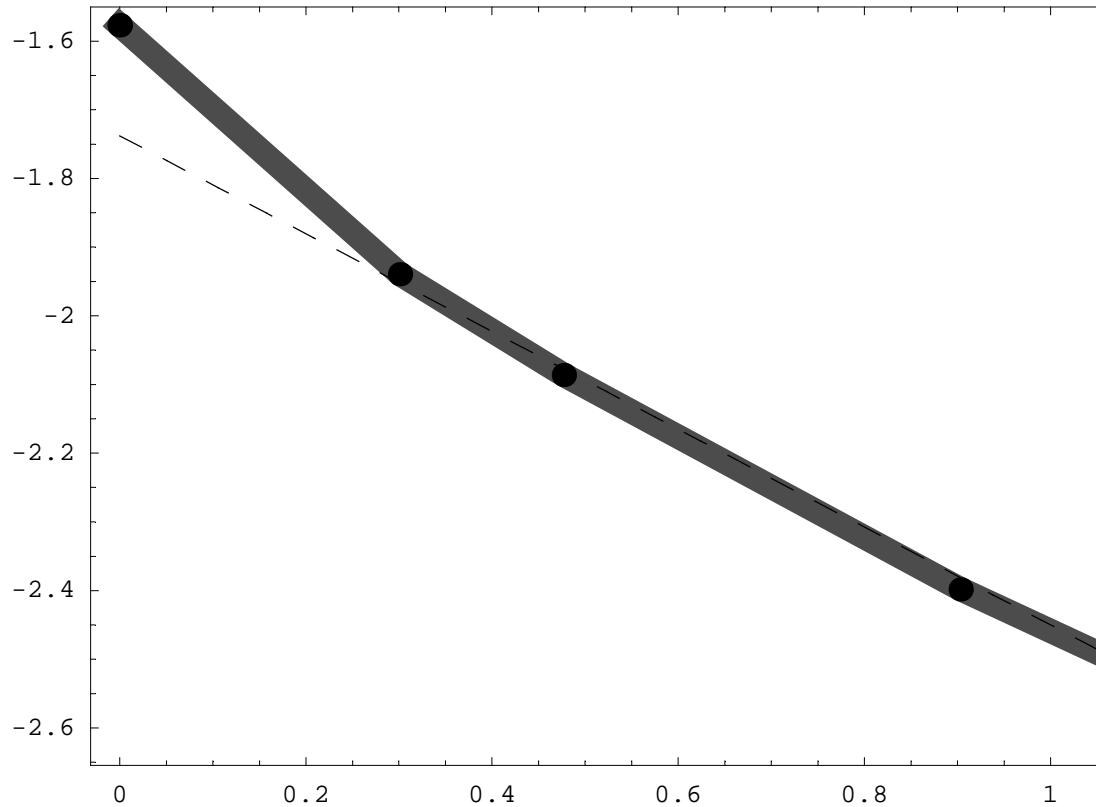
All the points except for the 3-click point fall fairly well onto a line. Now find the click count offset value which gives the closest shape to a straight line:

loglogjnds = Log[10, Map[{#[[1]] - 2.66, #[[2]]} &, jnds]] // N

$\{\{-0.468521, -1.57675\}, \{0.127105, -1.9393\},$
$\{0.369216, -2.08619\}, \{0.865696, -2.39794\}, \{1.23905, -2.61979\}\}$

```
p0 =
  ListPlot[loglogjnds, Frame → True, Axes → False, PlotJoined → True,
   PlotRange → All, PlotStyle → {Hue[0], Thickness[0.02]}];
```
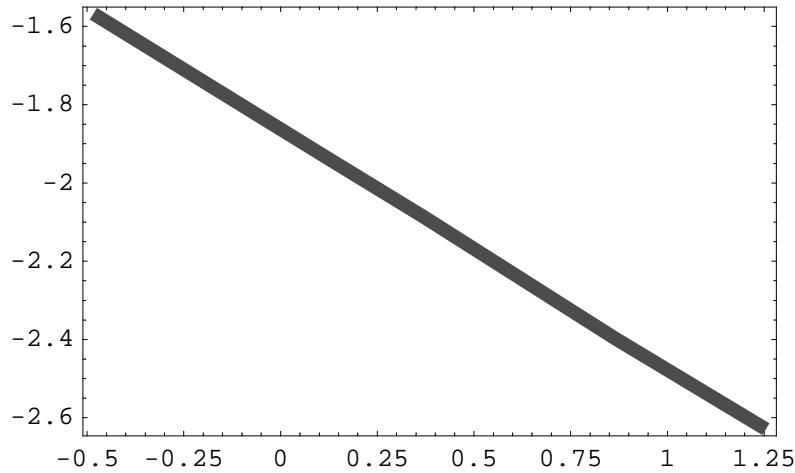


```
p1 = ListPlot[loglogjnds, Frame → True,
   Axes → False, PlotJoined → False, PlotRange → All,
   PlotStyle → {PointSize[0.02], Thickness[0.02]}];
```
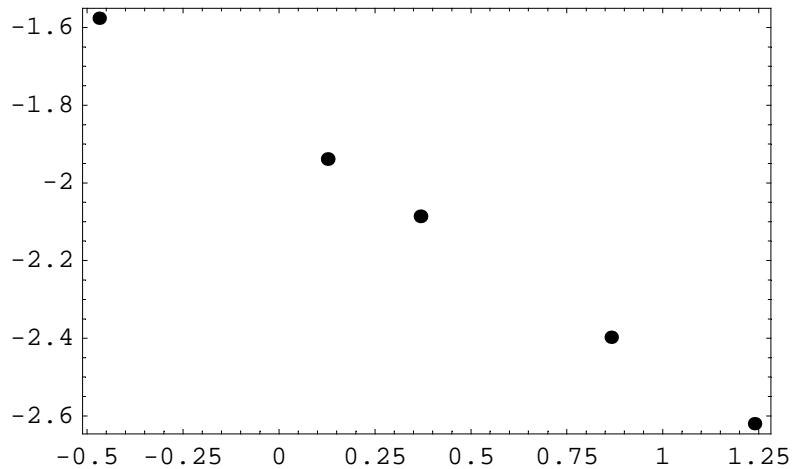


```
fit = Fit[loglogjnds, {1, x}, x]
```

$-1.86268 - 0.612684\,x$

```
fitp = Plot[fit, {x, -0.42, 1.25}, Frame → True,
   Axes → False, PlotStyle → Dashing[{0.02, 0.02}]];
```
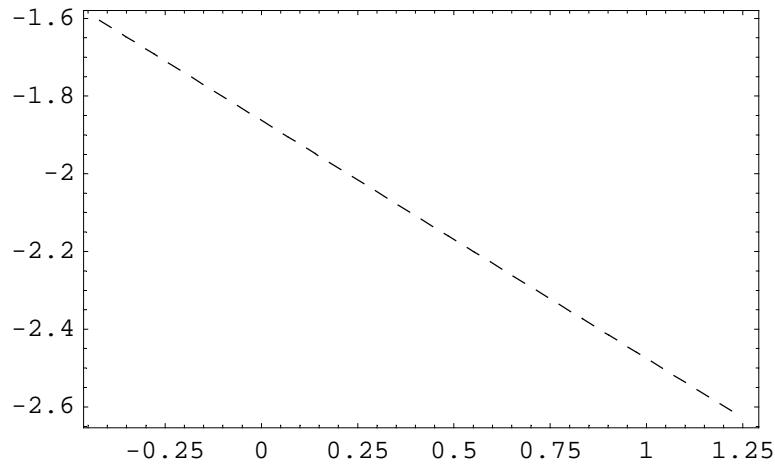


```
Show[p0, p1, fitp];
```
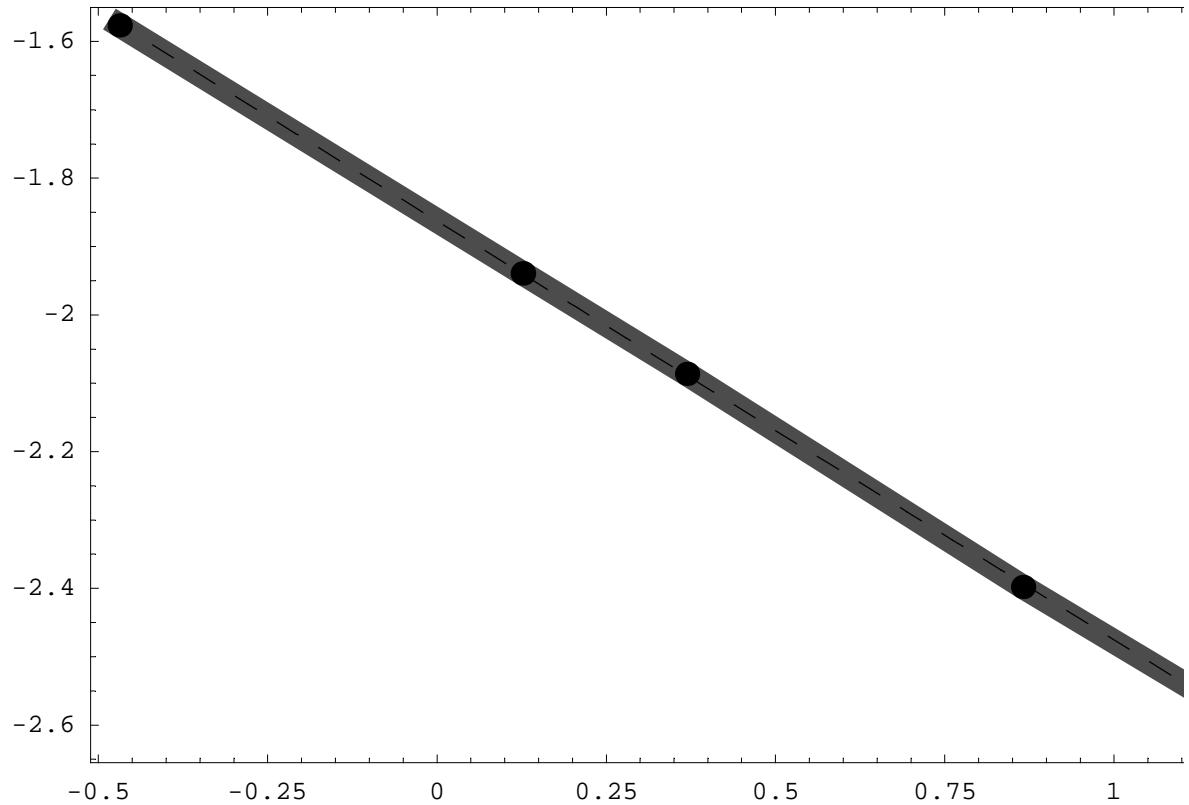
Calculating the absolute time difference at final click between constant tempo and changed tempo at the JND points.

```
clicks = {3 , 4 , 5 , 10, 20};
jnd = {0.0265, 0.0115, 0.0082 , 0.0040, 0.0024};
both = Transpose[{clicks, jnd}]
```

{{3, 0.0265}, {4, 0.0115}, {5, 0.0082}, {10, 0.004}, {20, 0.0024}}

```
constantends = Map[(1000 * (#[[1]] - 1)) &, both]
```

{2000, 3000, 4000, 9000, 19000}

```
slowertempos = Map[60000.0 / (1000 * (#[[2]] + 1) ^ (#[[1]] - 1)) &, both]
```

{56.9421, 57.9767, 58.0717, 57.8826, 57.3286}

```
fastertempos = Map[60000.0 / (1000 * (1 - #[[2]]) ^ (#[[1]] - 1)) &, both]
```

{63.311, 62.1185, 62.009, 62.2038, 62.8028}

```
slowerends = Map[Apply[Plus, #] &,
  Map[Table[1000.0 * (#[[2]] + 1)^n, {n, 0, #[[1]] - 2}] &, both]]
```

{2026.5, 3034.63, 4049.47, 9145.35, 19416.}

```
fasterends = Map[Apply[Plus, #] &,
  Map[Table[1000.0 * (1 - #[[2]])^n, {n, 0, #[[1]] - 2}] &, both]]
```

{1973.5, 2965.63, 3951.07, 8857.34, 18595.1}

Now calculate the difference between a steady tempo and a tempo increasing or decreasing at the measured JND rate:

```
fasterends - constantends
```

{-26.5, -34.3677, -48.9316, -142.664, -404.872}

```
slowerends - constantends
```

{26.5, 34.6323, 49.4695, 145.352, 416.035}

Plot the slower tempo offset data versus the click count:

```
dataslow = Transpose[{clicks, slowerends - constantends}]
```

$\{\{3, 26.5\}, \{4, 34.6323\}, \{5, 49.4695\}, \{10, 145.352\}, \{20, 416.035\}\}$

```
pointplot = ListPlot[dataslow,
    PlotStyle → PointSize[0.02], Axes → False, Frame → True];
```
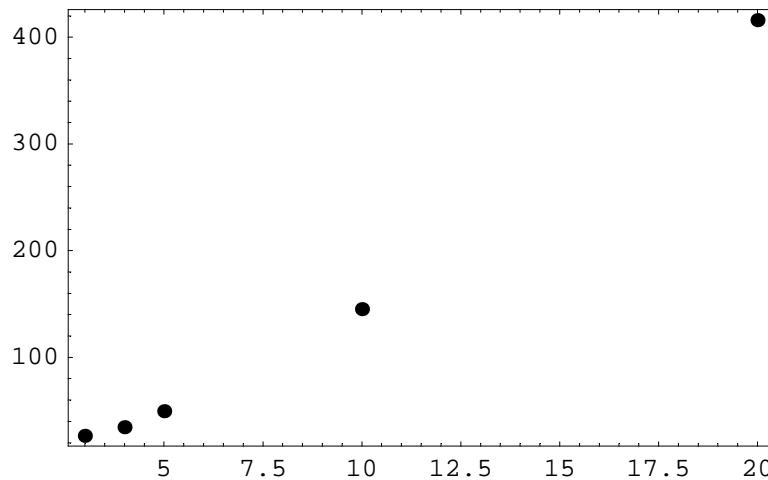


```
fit = Fit[dataslow, {1, x, x^2}, x]
```

$-10.8341 + 9.58046\, x + 0.588692\, x^2$

```
fitplot = Plot[fit, {x, 0, 20}, Frame → True];
```
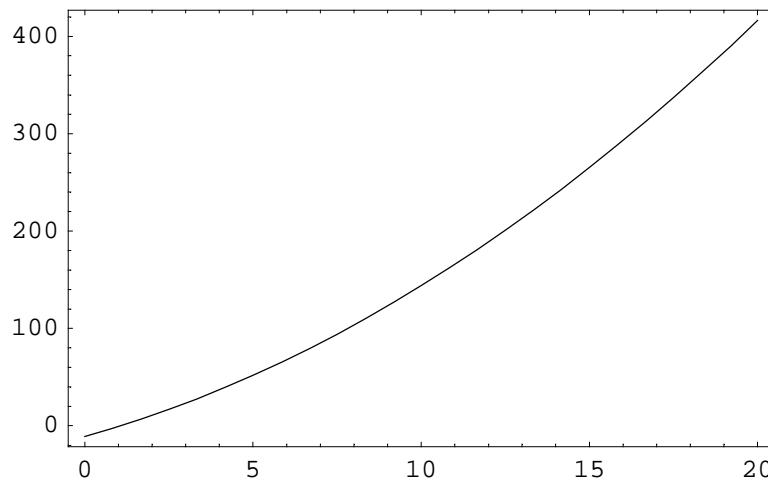

A parabolic cuve fits the data well with slight flattening at the bottom (cannot have a negative data point as the curve predicts on the far right). Anyway, it is not possible to go below 3 clicks for this type of test.

```
Show[pointplot, fitplot];
```
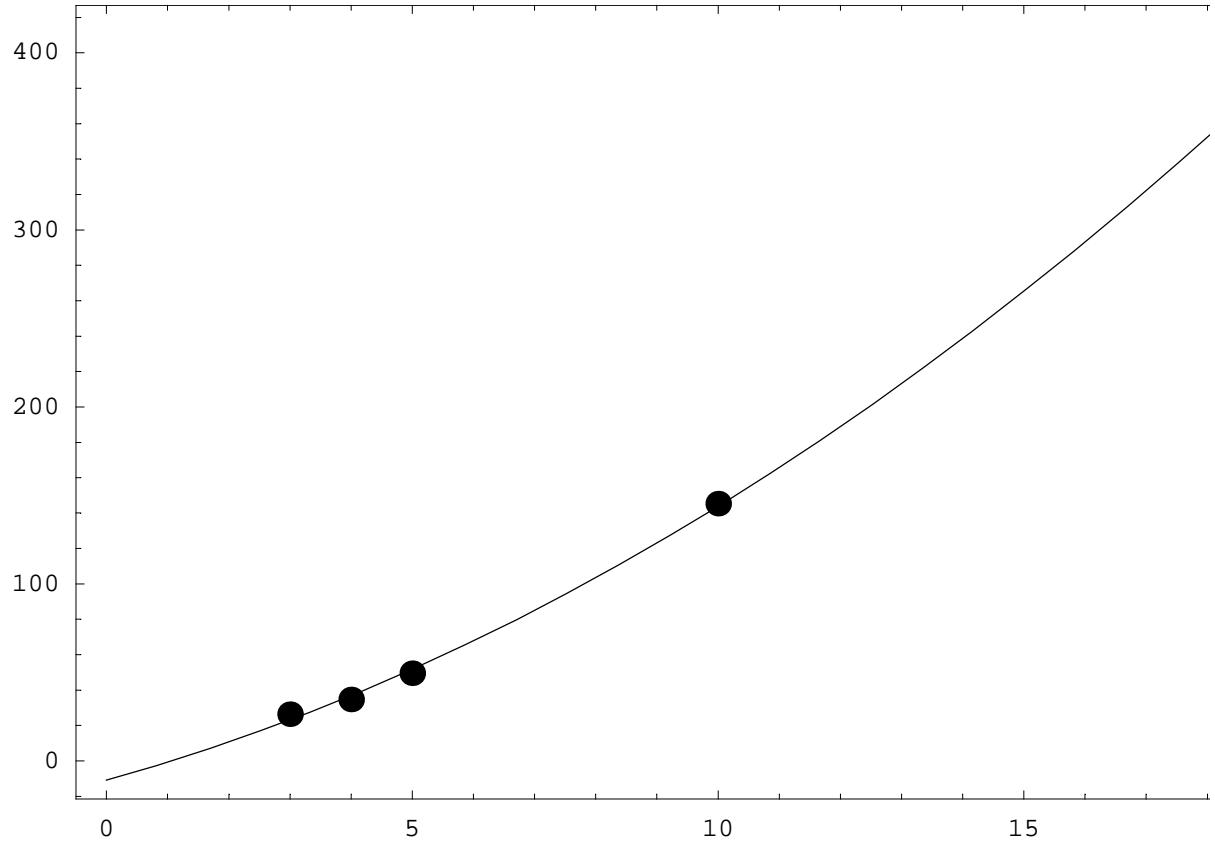

Plotting final tempo for JND rate versus click count

```
fastpoints = Transpose[{clicks, fastertempos}]
```

{{3, 63.311}, {4, 62.1185}, {5, 62.009}, {10, 62.2038}, {20, 62.8028}}

```
fpoints = ListPlot[fastpoints,
    Frame → True, Axes → False, PlotStyle → PointSize[0.02]];
```
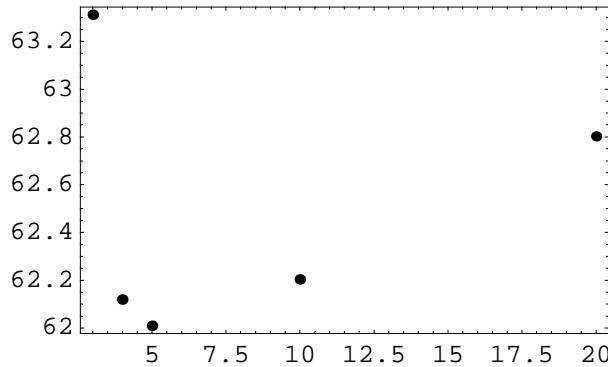


```
flines = ListPlot[fastpoints, Frame → True, Axes → False,
    PlotJoined → True, PlotStyle → {Hue[0], Thickness[0.02]}];
```
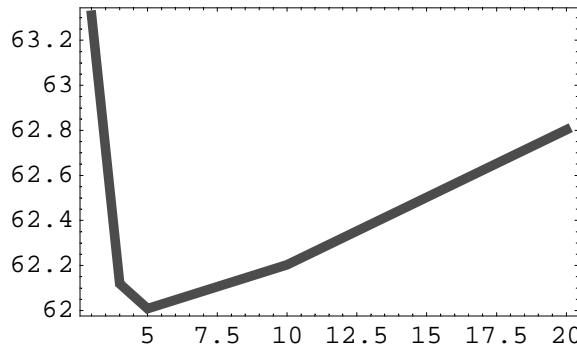


```
ffit = Fit[{fastpoints[[4]], fastpoints[[5]]}, {1, x}, x]
```

$61.6049 + 0.0598941\, x$

If the number of clicks in the test data is 1000, then the predicted final tempo at the JND would be about 120 MM. Since the starting tempo was 60 MM, it is probable that this estimate is too large.

```
ffit /. x → 1000
```

121.499

Here is an extra line through the 10 and 20 click points just for reference:

```
ffitplot = Plot[ffit, {x, 5, 20},
    PlotStyle → Dashing[{0.02, 0.02}], Frame → True, Axes → False];
```
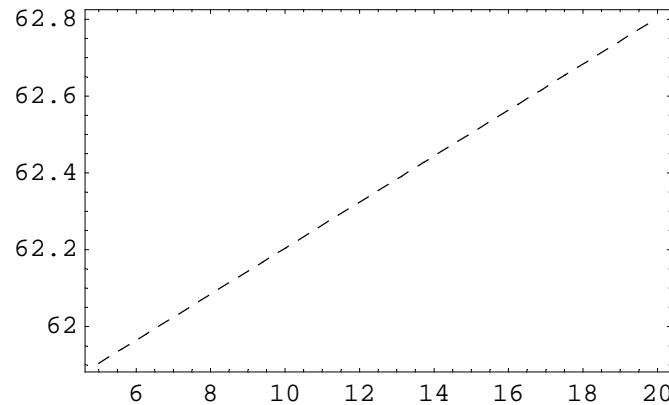


```
Show[flines, fpoints, ffitplot];
```
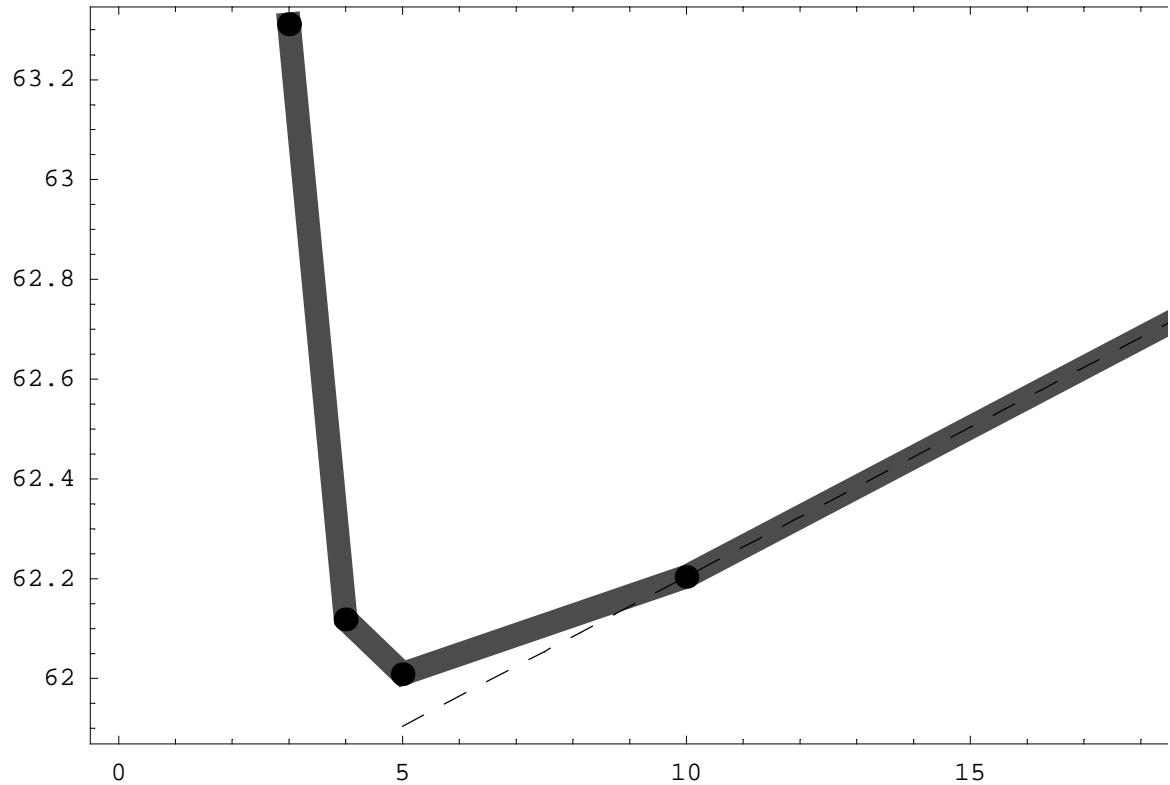

Calculating the ending tempos for various gains

Calculate the ending tempo at various gains for the 20 click sound example:

```
nn = 20;
string = "data" <> ToString[nn] <> "60";
tdata = ToExpression[string] // N;
min =
  Map[(60000 / (1000.0 (1 + #) ^ (nn - 1))) &, Transpose[tdata][[1]]];
max = Map[(60000 / (1000.0 (1 - #) ^ (nn - 1))) &, Transpose[tdata][[1]]];
table = Transpose[
    {Transpose[tdata][[1]], Transpose[tdata][[2]], min, max}] // N;
table // TableForm
```

|			
0.006	1.	53.5537	67.2682
0.005	1.	54.5753	65.9953
0.004	0.875	55.6174	64.7476
0.003	0.775	56.6805	63.5248
0.0024	0.85	57.3286	62.8028
0.002	0.625	57.765	62.3262
0.0015	0.7	58.3154	61.7359
0.001	0.55	58.8713	61.1515
0.0005	0.575	59.4328	60.5729
0.00025	0.525	59.7157	60.2857
0.000125	0.475	59.8577	60.1427
0.0000625	0.5	59.9288	60.0713

Calculate the ending tempo at various gains for the 10 click sound example:

```
nn = 10;
string = "data" <> ToString[nn] <> "60";
tdata = ToExpression[string] // N;
min =
  Map[(60000 / (1000.0 (1 + #) ^ (nn - 1))) &, Transpose[tdata][[1]]];
max = Map[(60000 / (1000.0 (1 - #) ^ (nn - 1))) &, Transpose[tdata][[1]]];
table = Transpose[
    {Transpose[tdata][[1]], Transpose[tdata][[2]], min, max}] // N;
table // TableForm
```

0.01	0.95	54.8604	65.6802
0.008	1.	55.8479	64.498
0.006	0.925	56.8551	63.3394
0.005	0.8	57.3663	62.7688
0.004	0.7	57.8826	62.2038
0.003	0.725	58.404	61.6446
0.002	0.675	58.9307	61.0909
0.001	0.625	59.4627	60.5427
0.0005	0.525	59.7307	60.2707

Calculate the ending tempo at various gains for the 5 click sound example:

```
nn = 5;
string = "data0" <> ToString[nn] <> "60";
tdata = ToExpression[string] // N;
min =
  Map[(60000 / (1000.0 (1 + #) ^ (nn - 1))) &, Transpose[tdata][[1]]];
max = Map[(60000 / (1000.0 (1 - #) ^ (nn - 1))) &, Transpose[tdata][[1]]];
table = Transpose[
    {Transpose[tdata][[1]], Transpose[tdata][[2]], min, max}] // N;
table // TableForm
```

0.016	0.975	56.3088	63.9987
0.014	0.95	56.7544	63.481
0.01	0.8	57.6588	62.4612
0.008	0.75	58.1178	61.959
0.006	0.625	58.5813	61.4619
0.004	0.575	59.0495	60.9697

Calculate the ending tempo at various gains for the 4 click sound example:

```
nn = 4;
string = "data0" <> ToString[nn] <> "60";
tdata = ToExpression[string] // N;
min =
  Map[(60000 / (1000.0 (1 + #) ^ (nn - 1))) &, Transpose[tdata][[1]]];
max = Map[(60000 / (1000.0 (1 - #) ^ (nn - 1))) &, Transpose[tdata][[1]]];
table = Transpose[
    {Transpose[tdata][[1]], Transpose[tdata][[2]], min, max}] // N;
table // TableForm
```

0.025	0.95	55.716	64.7347
0.02	0.9	56.5393	63.7489
0.015	0.786885	57.379	62.7831
0.012	0.8	57.8908	62.2129
0.01	0.725	58.2354	61.8366
0.008	0.875	58.5827	61.4634
0.006	0.675	58.9328	61.0931

Calculate the ending tempo at various gains for the 3 click sound example:

```
nn = 3;
string = "data0" <> ToString[nn] <> "60";
tdata = ToExpression[string] // N;
min =
  Map[(60000 / (1000.0 (1 + #) ^ (nn - 1))) &, Transpose[tdata][[1]]];
max = Map[(60000 / (1000.0 (1 - #) ^ (nn - 1))) &, Transpose[tdata][[1]]];
table = Transpose[
    {Transpose[tdata][[1]], Transpose[tdata][[2]], min, max}] // N;
table // TableForm
```

```
0.002     0.575     59.7607     60.2407
0.004     0.6       59.5229     60.4829
0.006     0.6       59.2864     60.7265
0.008     0.7       59.0514     60.9716
0.01      0.6       58.8178     61.2182
0.015     0.65      58.2397     61.8413
0.02      0.625     57.6701     62.474
0.025     0.775     57.1089     63.1164
0.03      0.75      56.5558     63.7687
0.035     0.8       56.0106     64.4313
0.04      0.775     55.4734     65.1042
0.045     0.95      54.9438     65.7877
0.05      0.975     54.4218     66.482
```